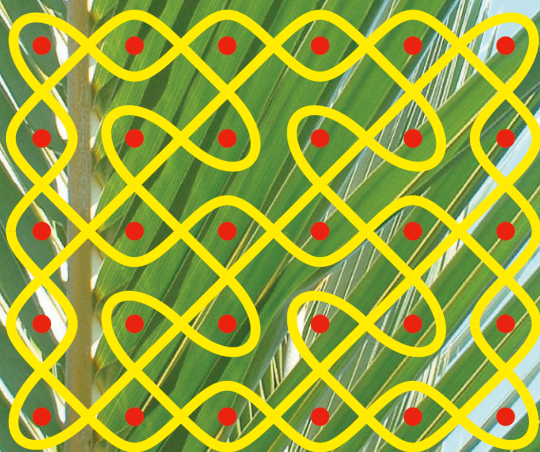


Paulus Gerdes

Lusona

Recreações
Geométricas
de África



Problemas e Soluções
(a cores)



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Paulus Gerdes

LUSONA
**RECREAÇÕES
GEOMÉTRICAS DE
ÁFRICA**

PROBLEMAS E SOLUÇÕES

ISTEG

Belo Horizonte, Boane

Moçambique

2012

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Problemas e Soluções

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PREFÁCIO 1

Se você tiver executado a maioria dos problemas, então, provavelmente, já conhece a maioria dos conceitos matemáticos referidos aqui. Este livro trata da matemática de uma maneira diferente, não apenas em termos de conteúdo, mas também em termos de estilo. O objetivo deste livro é apresentar a matemática de uma maneira que seja acessível a todos. O livro contém muitos exemplos e problemas, e a maioria deles é resolvida passo a passo. O livro também contém muitos exercícios e problemas para serem resolvidos pelo leitor. O livro é escrito em um estilo claro e direto, e a linguagem é simples e fácil de entender. O livro é adequado para estudantes de matemática em nível de graduação e para aqueles que desejam aprender matemática de uma maneira diferente. O livro é escrito por um autor que é um matemático experiente e que tem escrito muitos livros sobre matemática. O livro é um excelente recurso para quem deseja aprender matemática de uma maneira diferente.

The Mathematical Intelligencer”, embora, apelidado 'homem de São-Lucas', o seu trabalho tem sido reconhecido por alguns dos seus esforços, implacavelmente de Arq...
“O Turista Matemático”, na revista “The Mathematical Intelligencer”, embora, apelidado 'homem de São-Lucas', o seu trabalho tem sido reconhecido por alguns dos seus esforços, implacavelmente de Arq...
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Redator da coluna “O Turista Matemático” na revista “The Mathematical Intelligencer”

Doutor Dirk Huylebrouck

Departamento de Arquitetura São-Lucas, Bélgica.

Redator da coluna “O Turista Matemático” na revista

“The Mathematical Intelligencer”

PREFÁCIO 2

A Matemática está à nossa volta, em toda a parte.

Na Natureza, no desenvolvimento econômico, científico, tecnológico, mas também intimamente ligada à cultura e às tradições. Embora muitas vezes não seja óbvia a presença da Matemática, ou a razão de ser dessa presença, se observarmos com alguma atenção, podemos (re)descobrir essa presença matemática e conjecturar as bases da sua presença.

Os *Lusona* dos Quiocos (Cokwe) de Angola e Congo são fascinantes manifestações de técnicas matemáticas que nos podem levar a explorações muito ricas. O autor, Paulus Gerdes, que há mais de 20 anos se dedica à análise da matemática (“congelada” segundo os seus próprios termos) nas tradições culturais de várias civilizações (sobretudo africanas) propõe-nos com este livro uma descoberta e um desafio. Vamos responder a esse desafio e propor mais padrões de acordo com as regras dos *Lusona*? Mãos à obra!

Coimbra, aos 10 de Março de 2001

Professor Jaime Carvalho e Silva
Departamento de Matemática
Universidade de Coimbra

(atualmente 2010: Secretário-Geral da Comissão Internacional para
Educação Matemática)

APRESENTAÇÃO DA SEGUNDA EDIÇÃO

Os desenhos tradicionais dos Cokwe de Angola, chamados *(lu)sona*, fascinaram-me desde que os vi pela primeira vez. Logo fiquei com a impressão de que se tratava tanto duma bela escrita geométrica como duma fala artística.

Aprendi a executar *(lu)sona*, iniciei o estudo da geometria por detrás dos *(lu)sona*, e comecei a experimentar as possibilidades de explorar os *(lu)sona* na educação matemática.

Um dos primeiros frutos dessa experimentação didática foi um pequeno livro publicado na série brasileira *Vivendo a Matemática* sob o título de *Desenhos da África*. Trata-se dum livro elaborado para crianças na faixa etária de 8 a 14 anos, que dá uma breve introdução à cultura Cokwe, aos *sona*, e, através duma série de atividades, estuda temas como o ritmo dos desenhos (introduzindo o conceito de algoritmo) e a simetria axial e rotacional dos desenhos, e aproveita os *sona* para obter ideias de como calcular algumas somas interessantes (levando a uma introdução a sucessões aritméticas). Centenas de crianças escreveram-me a partir do Brasil; às vezes turmas inteiras enviaram-me novos desenhos inspirados pelos *(lu)sona* dos Cokwe de Angola. A Fundação Brasileira para o Livro Juvenil premiou o livro como um dos melhores livros juvenis de divulgação científica publicado no Brasil em 1990.

Em 1991 publiquei, em Moçambique, uma coletânea de divertimentos geométricos para alunos do ensino secundário. A coletânea foi o fruto de experimentação com um grupo de alunos secundários e dum grupo de estudantes da formação de professores de matemática na cidade de Maputo. O livro chamava-se *Lusona: Recreações Geométricas de África*, que agora se reedita. Nele se apresentam vários problemas em que se mostram algumas figuras de tamanhos diferentes, mas construídas seguindo uma mesma regra. A

tarefa consiste em descobrir outras figuras do mesmo tipo que faltam na série apresentada. Todas as figuras dadas são ou *(lu)sona* tradicionais ou novas figuras criadas pelo autor, mas inspiradas pela tradição Cokwe, quer dizer, as novas figuras são no mesmo estilo que os *(lu)sona*.

Um professor do ensino médio ou secundário pode utilizar o livro ***Lusona: Recreações Geométricas de África***, copiando um ou mais problemas para cada aluno ou mostrando um problema em transparência com a ajuda dum retroprojektor, e pedir aos alunos para desenhar as figuras apresentadas, em papel quadriculado (ou em papel de pontos equidistantes) e descobrir as figuras que faltam. A seguir, os alunos podem analisar em grupo quais são as ideias que desenvolveram durante a resolução do(s) problema(s). Em casa ou numa outra aula poder-se-á apresentar mais recreações.

Na segunda parte do livro estimula-se o leitor a inventar as suas séries de *(lu)sona* e outras séries inspiradas pelo estilo Cokwe.

Outros exemplos da exploração educacional dos *sona* apresentam-se no segundo volume do meu livro ***Geometria Sona***, inicialmente publicado em 1993. O primeiro volume da ***Geometria Sona de Angola: Matemática dum Tradição Africana*** dedica-se à reconstrução e à análise de elementos matemáticos na tradição *sona*, enquanto o terceiro volume apresenta uma análise comparativa com tradições doutros tempos e doutros povos que têm alguma semelhança com a tradição *(lu)sona*, como algumas tradições do Egito Antigo, da Mesopotâmia Antiga, de mulheres Tamil no Sul da Índia, dos Celtas, etc. A exploração do potencial matemático dos *(lu)sona* levou-me a publicar vários artigos científicos e à elaboração do livro ***Geometria Lunda*** (1996). Hoje em dia já há matemáticos em vários continentes que se dedicam ao estudo de ideias matemáticas que têm a sua origem na reflexão sobre a geometria por detrás dos *(lu)sona*.

Expresso o desejo de que os leitores da nova edição do livro ***Lusona: Recreações geométricas de África*** se sintam tão fascinados pelos *(lu)sona* como eu próprio e os meus alunos.

Expresso também o desejo de que os leitores se sintam, através dos *(lu)sona*, atraídos pelo aspecto lúdico da matemática e que aumentem o interesse pela matemática em geral, e pelo rico património científico das culturas africanas.

Divirtam-se!

Maputo, aos 27 de Fevereiro de 2001

Paulus Gerdes
Centro de Investigação Etnomatemática –
Cultura, Educação, Matemática
C. P. 915
Maputo
Moçambique

APRESENTAÇÃO DA TERCEIRA EDIÇÃO

Para além dos 90 recreações, problemas e desafios, a terceira edição contém também soluções.

É a primeira edição a cores.

INTRODUÇÃO

O leitor certamente conhece problemas aritméticos do tipo

“Encontre os números que faltam”,

onde alguns números duma sequência são dados e se devem procurar outros.

Por exemplo:

2 4 6 ? 10 12 ? ?

Por se tratar obviamente da sequência dos números pares, torna-se fácil preencher os números que faltam:

2 4 6 **8** 10 12 **14 16**, etc.

Outro exemplo:

24 34 46 60 ? ?

Desta vez será um pouco mais difícil encontrar os números que faltam. Tem de ser descoberta uma regra para obter (ou construir) os elementos desta série:

$$24 = 5 \times 5 - 1$$

$$34 = 6 \times 6 - 2$$

$$46 = 7 \times 7 - 3$$

$$60 = 8 \times 8 - 4$$

Uma vez encontrada esta regra de construção, não será difícil obter os elementos seguintes da sequência:

$$9 \times 9 - 5 = 76,$$

$$10 \times 10 - 6 = 94,$$

$$11 \times 11 - 7 = 114, \text{ etc.}$$

Deste modo, acha-se

$$24 \quad 34 \quad 46 \quad 60 \quad 76 \quad 94 \quad 114, \text{ etc.}$$

Observando que

$$34 - 24 = 10$$

$$46 - 34 = 12$$

$$60 - 46 = 14$$

$$76 - 60 = 16,$$

etc.,

pode-se encontrar mais uma regra de construção para esta mesma série numérica.

Na primeira parte (Capítulo 2) de *Lusona*, problemas do tipo

“Encontre as figuras que faltam”,

onde algumas figuras duma série são dadas e se deve achar outras.

Tal como no caso dos problemas do tipo “Encontre os números que faltam”, deve-se descobrir a regra de construção para os padrões dados e, então, desenhar as figuras que faltam, aplicando a mesma regra de construção.

Os padrões apresentados são no estilo de desenhos africanos tradicionais, chamados *sona*. O singular de *sona* é *lusona* na língua Cokwe. Cada um deles é composto por uma única linha (fechada) que se cruza muitas vezes consigo mesma e que abraça todos os pontos duma rede de pontos.

Na segunda parte do livro (Capítulo 3), dá-se um padrão de cada vez e convida-se o leitor a construir a sua própria série em que o padrão dado aparece como um dos elementos.

Lusona: Recreações Geométricas de África

Finalmente (Capítulo 5), o leitor é convidado a inventar os seus próprios *(lu)sona* e sequências de *(lu)sona* e a enviar as suas propostas ao autor. As propostas mais interessantes serão incluídas num segundo livro de recreações geométricas.

1 de Maio de 1990

Paulus Gerdes

C. P. 915, Maputo, Moçambique

CAPÍTULO 1

INFORMAÇÃO GERAL SOBRE A TRADIÇÃO DOS (LU)SONA

A tradição dos *(lu)sona* pertence à herança cultural dos povos Cokwe (ou Quiocos na forma aportuguesada), Lunda, Luena, Xinge e Minungo, que habitam o nordeste de Angola (África Austral). Especialmente os Cokwe são bem conhecidos pela sua arte decorativa que abrange desde a ornamentação de esteiras e cestos entrançados, trabalho em ferro, cerâmica, gravação de cabaças e tatuagens até pinturas nas paredes das casas e desenhos na areia.

Quando os Cokwe se encontram no centro da aldeia ou no acampamento de caça, sentados à volta da fogueira ou à sombra de árvores frondosas, costumam passar o tempo em conversas, ilustrando-as com desenhos na areia. A estes desenhos chamam-se *lusona* (singular) ou *sona* (plural). Muitos destes desenhos pertencem a uma velha tradição. Referem-se a provérbios, fábulas, jogos, animais, etc. e desempenham um papel importante na transmissão do conhecimento e da sabedoria de uma geração para a seguinte.

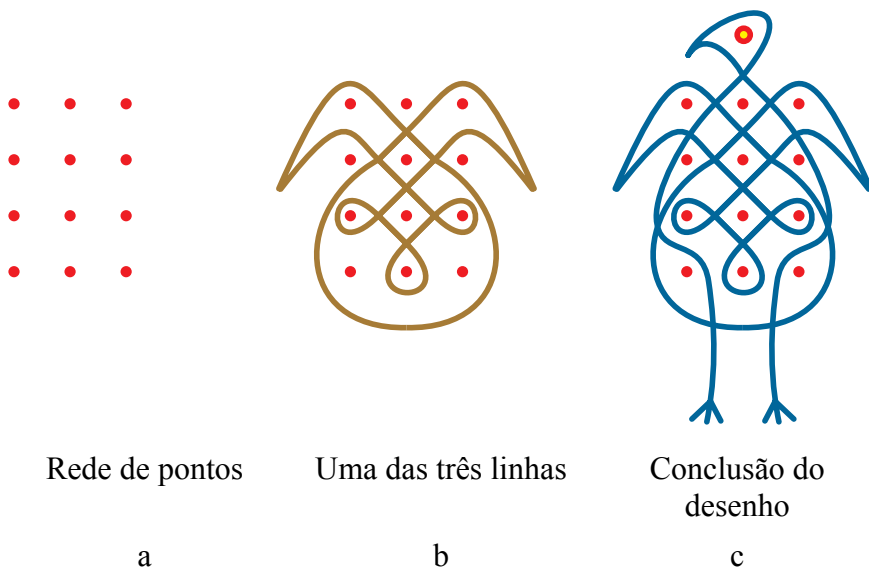
Cada rapaz aprende o significado e a execução dos desenhos mais simples durante a fase intensiva ‘escolar’ da circuncisão e ritos de iniciação. O significado e a feitura dos mais difíceis é transmitido por especialistas, os chamados *akwa kuta sona* (aqueles que sabem desenhar) a familiares mais novos.

Para desenhar um *lusona*, os Cokwe limpam e alisam o chão em primeiro lugar e depois marcam com as pontas dos dedos uma rede de pontos como esta:



Figura 1

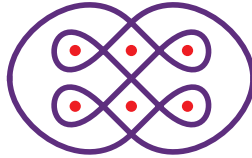
As distâncias entre dois pontos horizontal ou verticalmente vizinhos são todas iguais. O número de filas e o número de colunas da rede de pontos dependem do motivo a ser representado. Por exemplo, para representar um galo são precisas quatro filas de três pontos:



Representação dum galo
Figura 2

Este *lusona* é composto por três linhas (não tomando em conta os dedos do galo).

Sona compostos por uma única linha e simétricos eram muito populares no seio dos Cokwe. A representação das patas dum antílope serve de exemplo:



Patas de antílope

Figura 3

Com a penetração e ocupação coloniais, a tradição dos *(lu)sona* entrou em decadência. Alguns missionários e etnógrafos recolheram *sona* e livraram-nos do esquecimento (Fontinha, Hamelberger, Kubik, Pearson, Dos Santos,). A maior coleção de *sona* foi publicada por Fontinha. O seu livro contém 287 *sona* diferentes recolhidos nos anos 40 e 50. Já nessa altura foi difícil recolher muitos *sona*. Eram raras as pessoas que conheciam mais de uma meia dúzia de *sona*; só alguns velhos conheciam o “segredo” dos desenhos mais complicados e não era fácil convencê-los a executar estes *sona* por dizerem que embora antes tivessem sido hábeis nesta arte, já não tinham cabeça nem vista para os fazerem. Aqui é de salientar que os desenhos deviam ser executados suave e continuamente, pois hesitar ou parar a meio era tido como uma imperfeição, o que levaria os circunstantes a esboçar sorrisos irônicos.

No livro *Geometria Sona de Angola: Matemática duma Tradição Africana* tentei reconstruir o conhecimento matemático dos especialistas Cokwe de desenho e mostrar como esta velha tradição dos *sona* pode ser valorizada, enriquecida e desenvolvida.

O livro *Lusona: Recreações Geométricas de África* apresenta problemas geométricos inspirados na tradição *sona*. Espero que o leitor venha a gostar tanto de desenhar e inventar *sona* como eu próprio e os meus alunos.

CAPÍTULO 2

ENCONTRE AS FIGURAS EM FALTA

2.1 Primeiro exemplo

No caso do exemplo na Figura 1, o segundo e terceiro elementos de uma série de desenhos são dados. A tarefa consiste em encontrar o primeiro, quarto e quinto elementos da sequência, representados por sinais de interrogação.

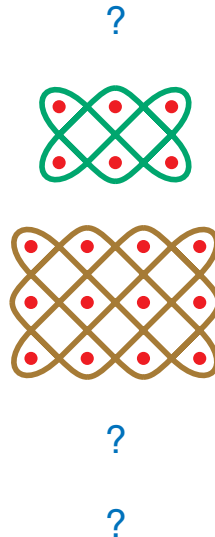


Figura 1

Ambos os padrões dados se desenham da mesma maneira. Cada um deles é composto por uma linha fechada, que tanto antes como

depois de uma viragem faz ângulos de 45° (metade de um ângulo recto) com os lados da rede de pontos (Figura 2).

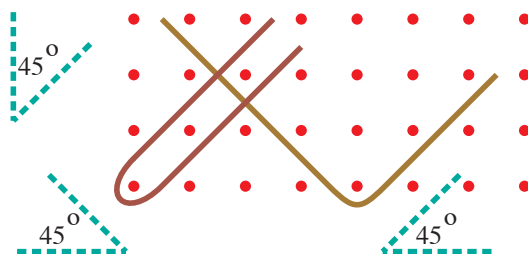


Figura 2

Para “sentir a regra nos dedos”, copie os padrões em papel transparente.

Em seguida, desenhe ambos os padrões em papel quadriculado, marcando em primeiro lugar a rede de pontos. Para o segundo elemento precisam-se de duas filas de três pontos cada uma; para o terceiro elemento, três filas de quatro pontos (Figura 3).

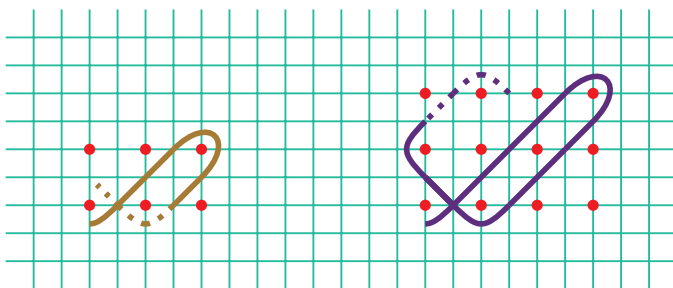


Figura 3

A continuação natural será:

- * Para o quarto elemento são precisas quatro filas de cinco pontos;
- * Para o quinto elemento precisam-se de cinco filas de seis pontos, etc.

Desenhe o quarto elemento (Figura 4).

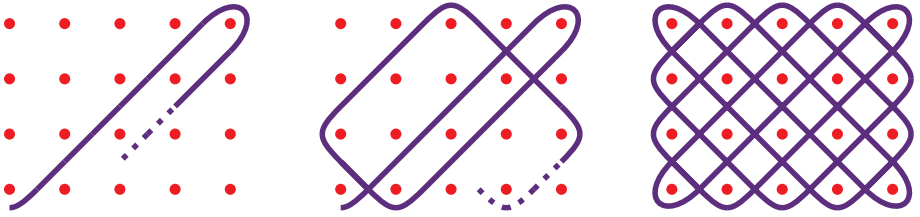


Figura 4

Desenhe o quinto elemento (Figura 5).

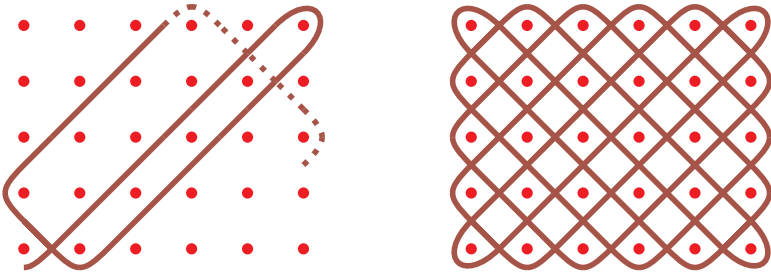


Figura 5

E o primeiro elemento? Como ficará?

Para o primeiro elemento precisa-se duma fila de dois pontos e ao aplicar a mesma regra de construção, obtém-se o primeiro elemento da série (Figura 6).



Figura 6

2.2 Segundo exemplo

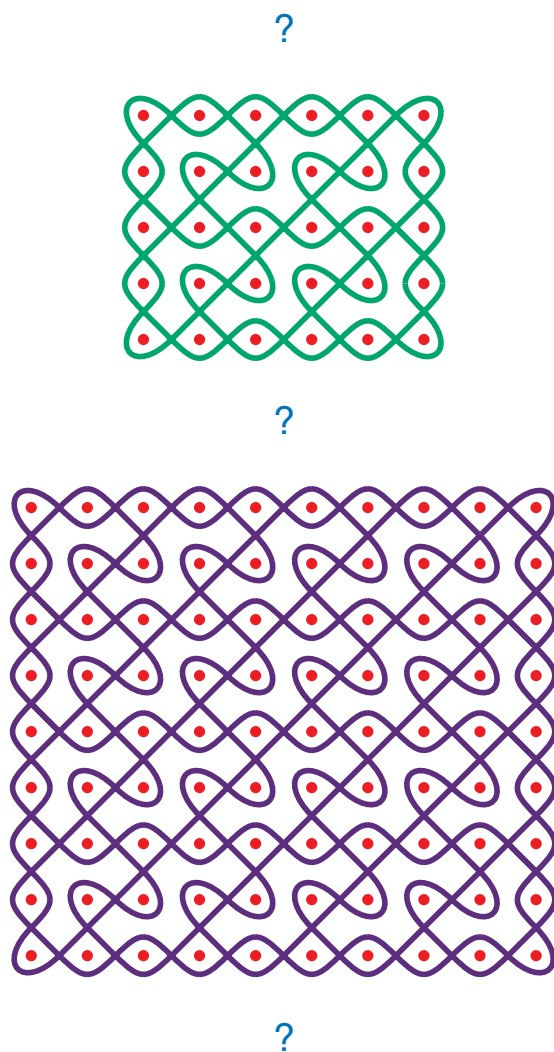


Figura 7

O segundo e o quarto elemento da série na Figura 7 são *sona* da tradição Cokwe. Representam as marcas deixadas no chão por um galo de mato ao ser perseguido. O leitor vê os ziguezagues consecutivos do galo para poder escapar?

No caso do segundo elemento, a rede de pontos é composta por cinco filas de seis pontos. No caso do quarto elemento, a rede é composta por nove filas de dez pontos.

O número de pontos em cada fila é igual ao número de filas mais um. Por sua vez, o número de filas é igual ao dobro do número do elemento mais um:

$$5 = 2 \times 2 + 1 \text{ para o segundo elemento e}$$

$$9 = 2 \times 4 + 1 \text{ para o quarto elemento.}$$

Agora pode-se adivinhar que para o primeiro elemento da série se precisa de $2 \times 1 + 1$, ou seja, de três filas (de quatro pontos). Do mesmo modo, para o terceiro elemento precisa-se de $2 \times 3 + 1$, ou seja, de sete filas (de oito pontos), e, para o quinto elemento, de $2 \times 5 + 1$, ou seja, de 11 filas de 12 pontos

Chegamos ao momento de podermos marcar as redes de pontos.

Para poder desenhar o primeiro, terceiro e quinto elementos da série, falta apenas conhecer a regra de construção.

Copie o segundo elemento em papel transparente, começando a execução do desenho pelo primeiro ziguezague à direita (Figura 8):

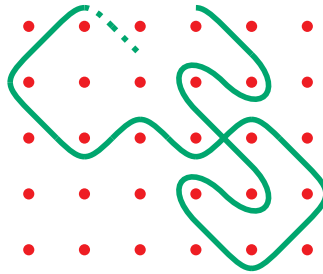
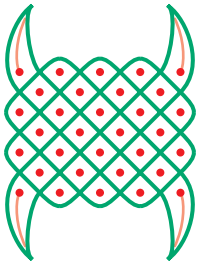


Figura 8

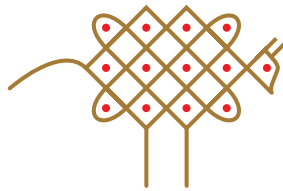
Copie da mesma maneira o quarto elemento (Figura 9).

Exemplos de SONA tradicionais

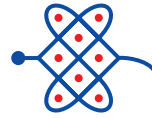
1



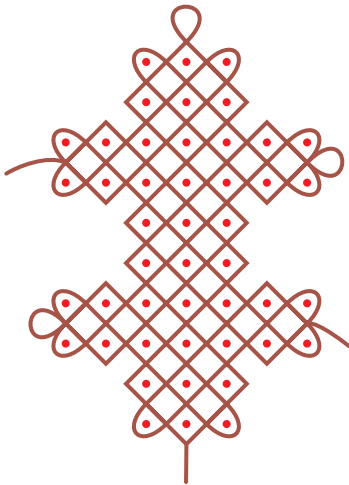
cabeça de búfalo



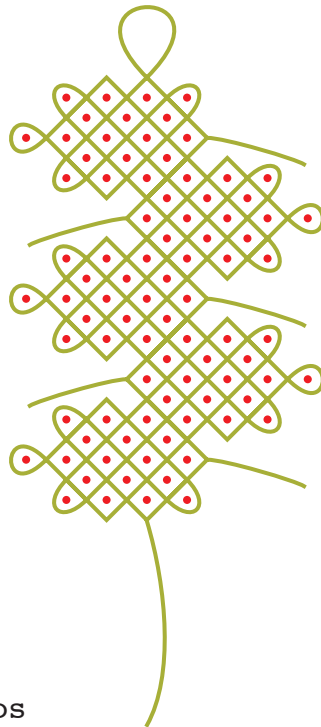
antílope



rato



leoa com dois filhos

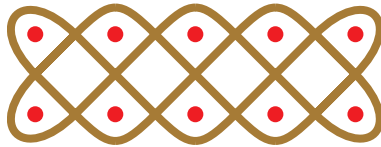


leopardo com cinco filhos

1



?

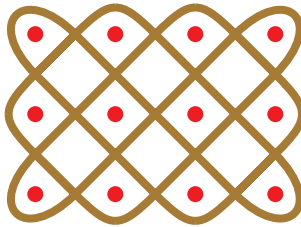
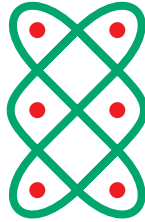


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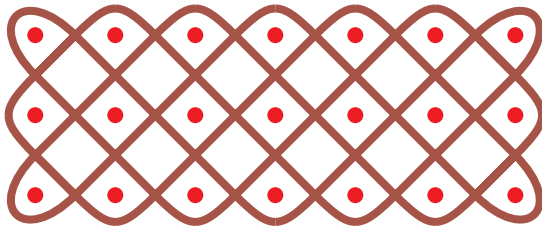
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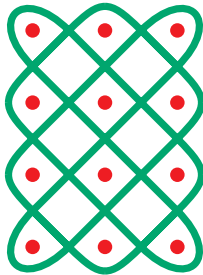
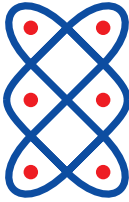


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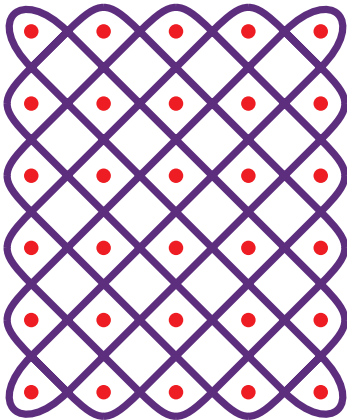


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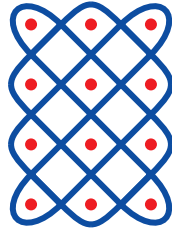


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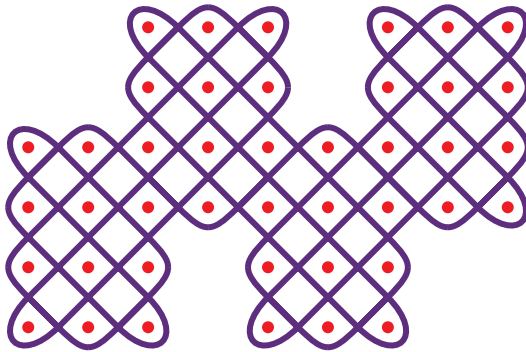
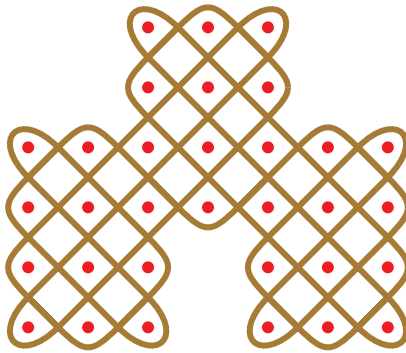


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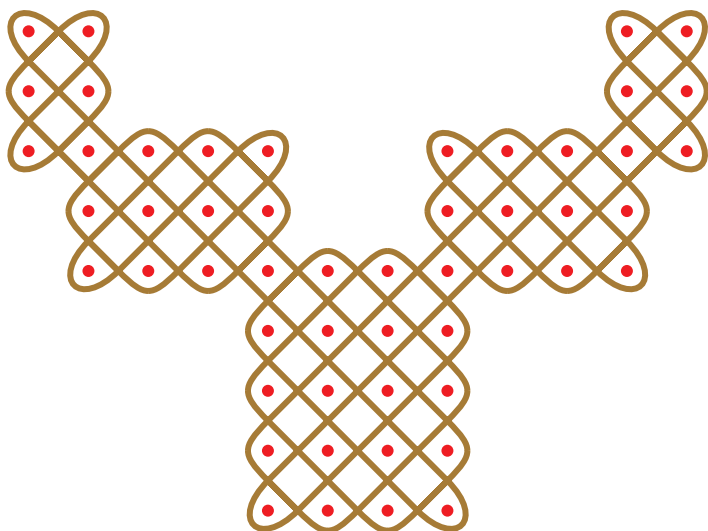
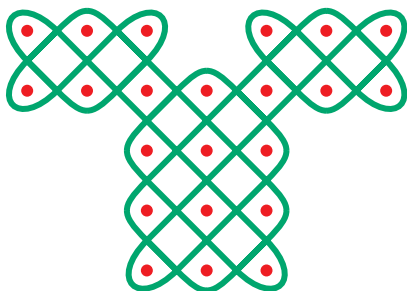


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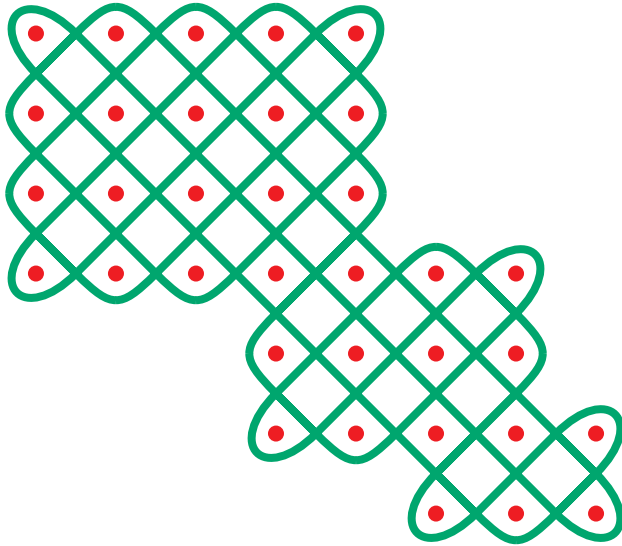
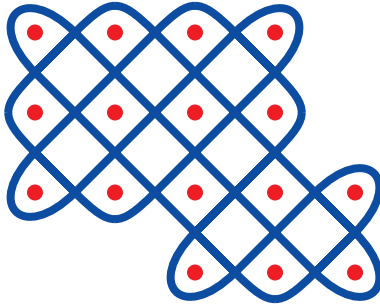
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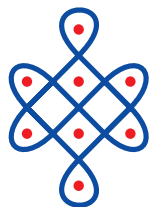
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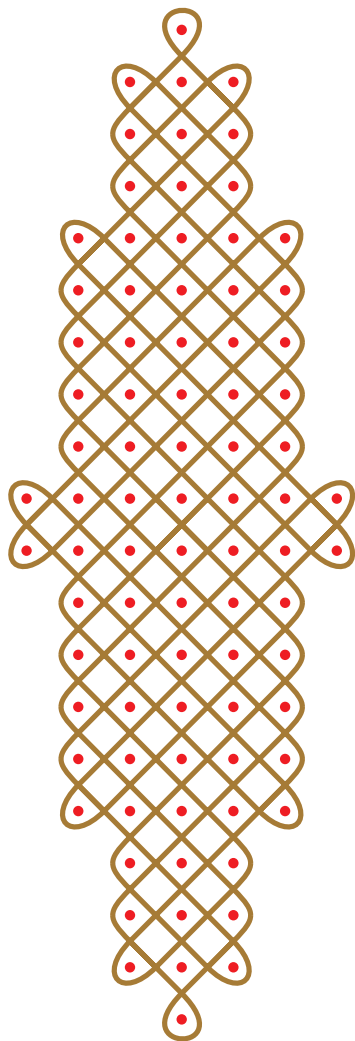
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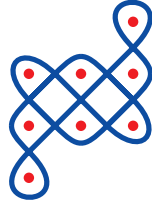


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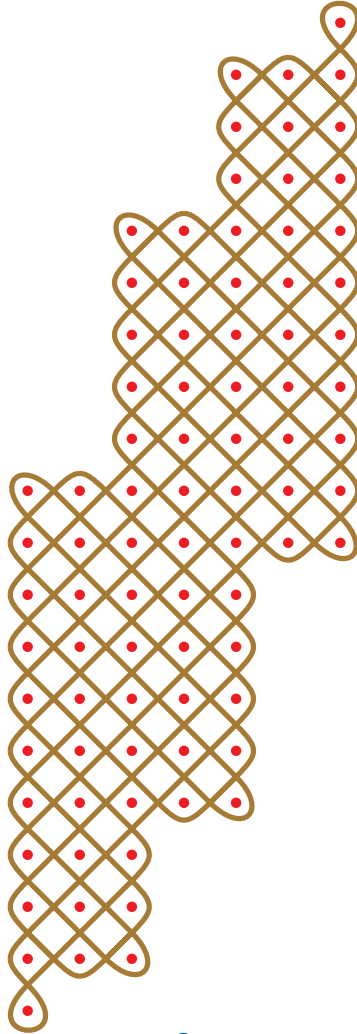


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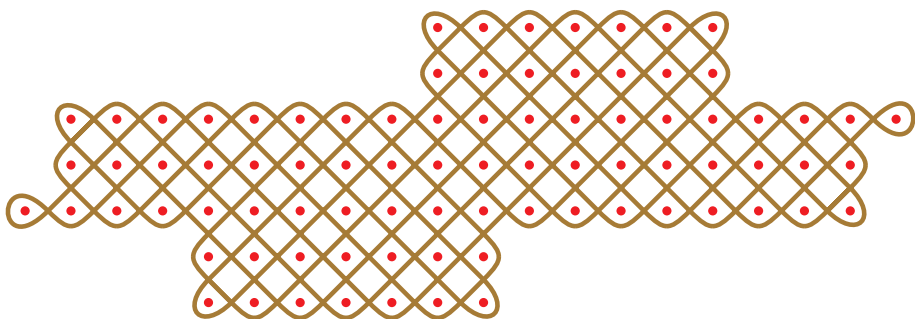
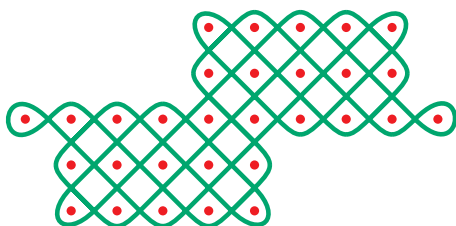
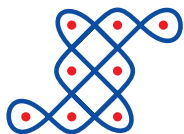
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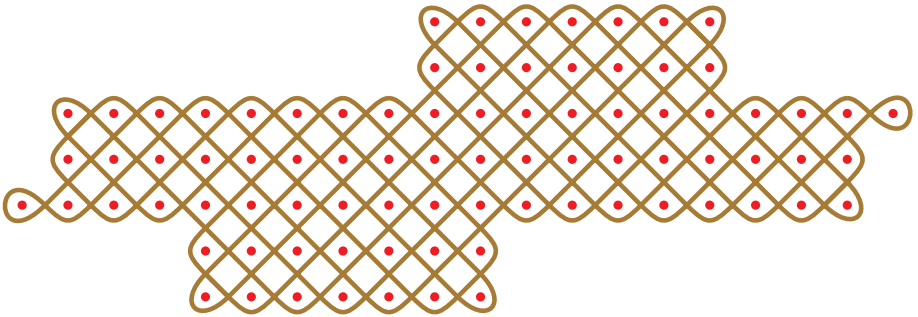
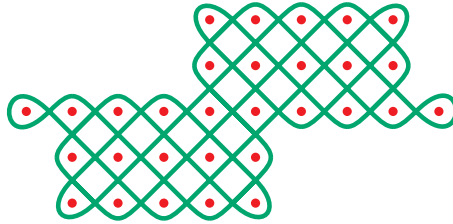
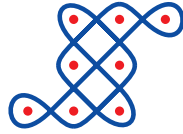
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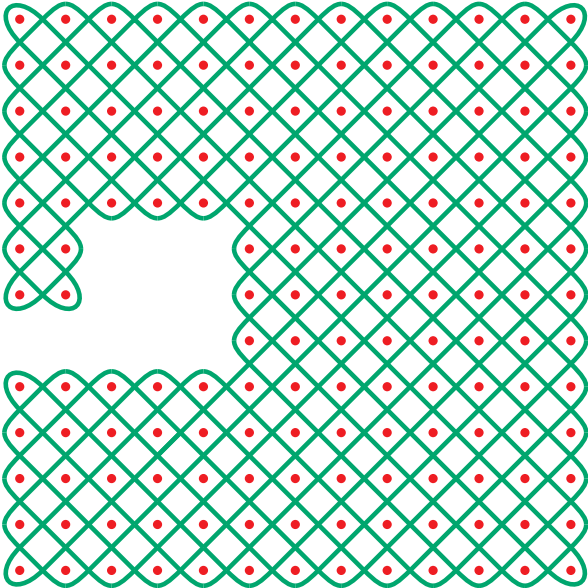
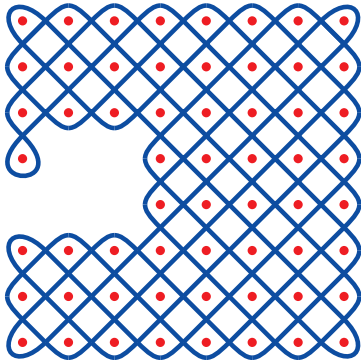
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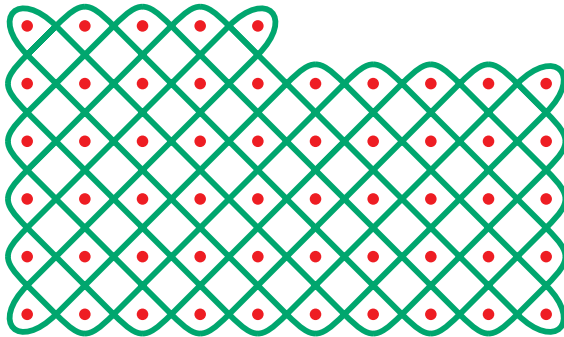
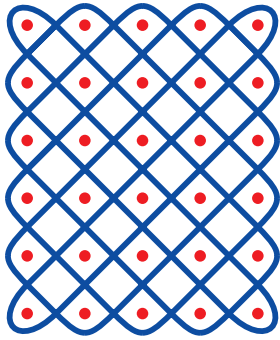
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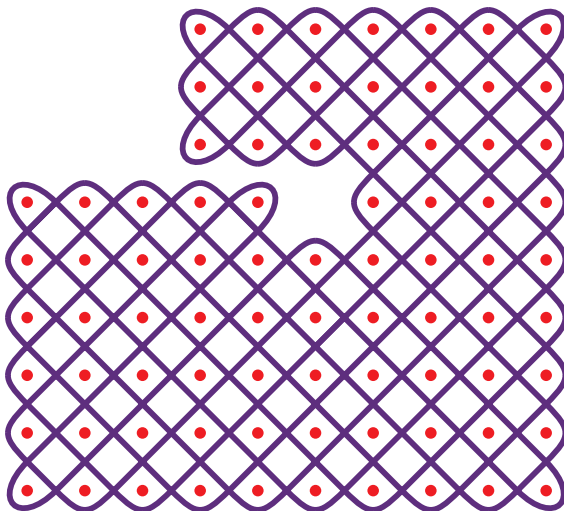


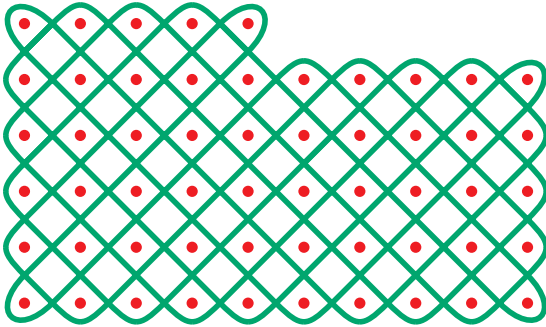
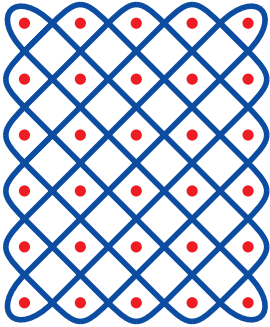
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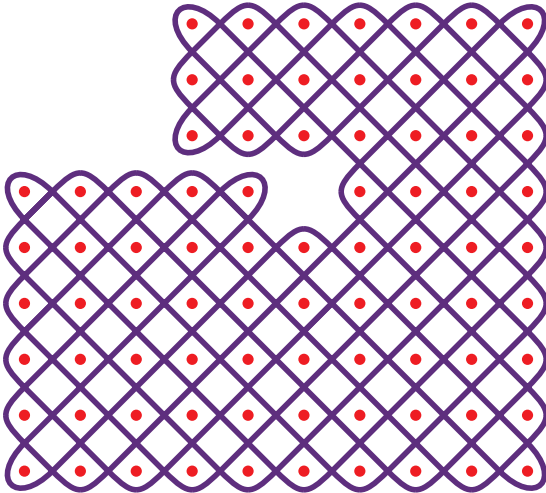


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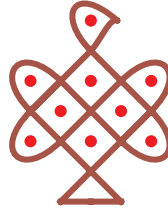


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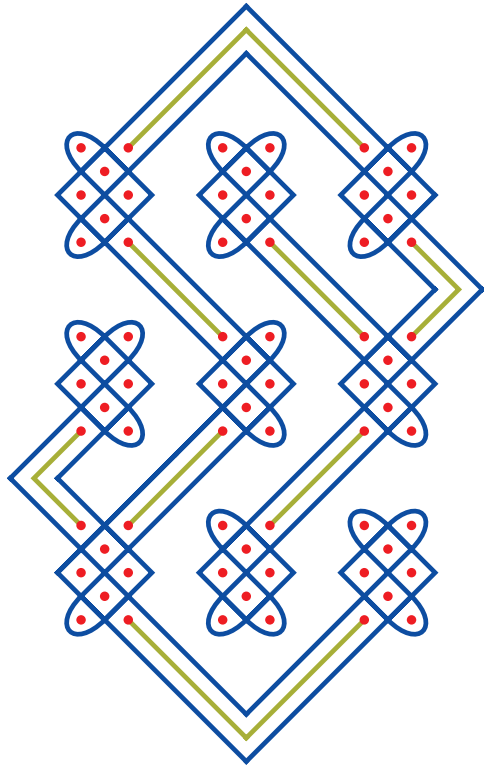
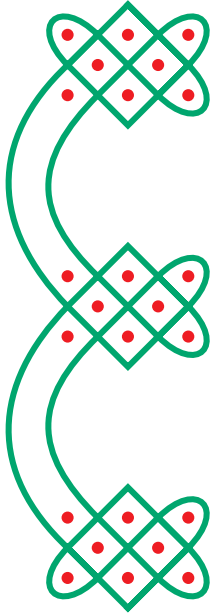


Exemplos de SONA tradicionais

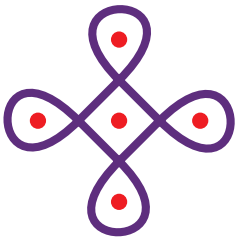
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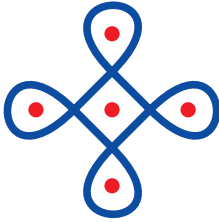


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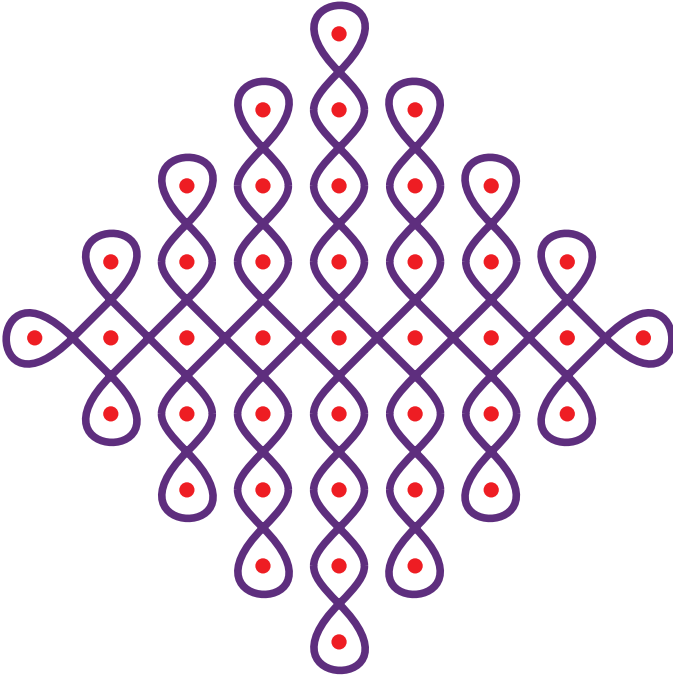
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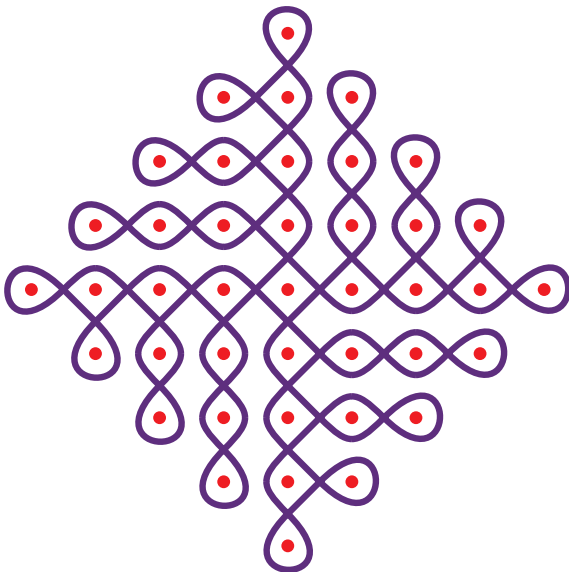
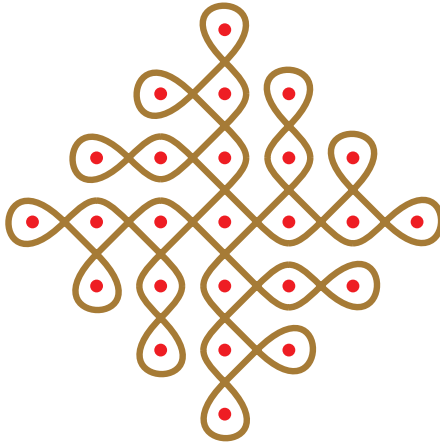


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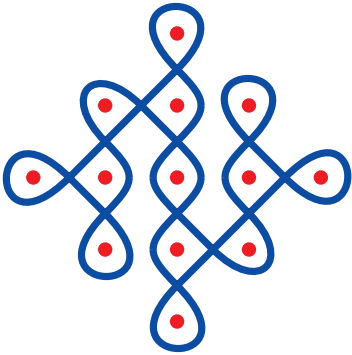
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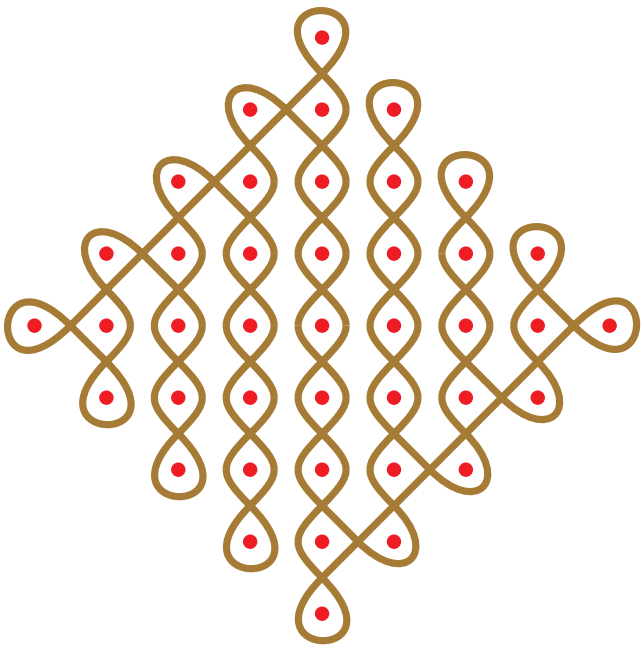


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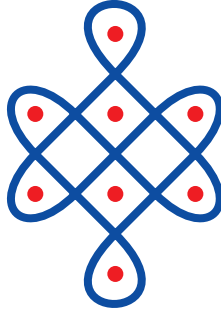
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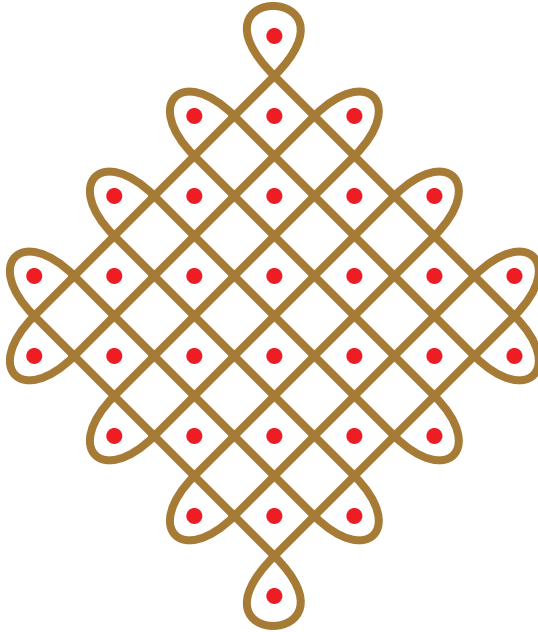
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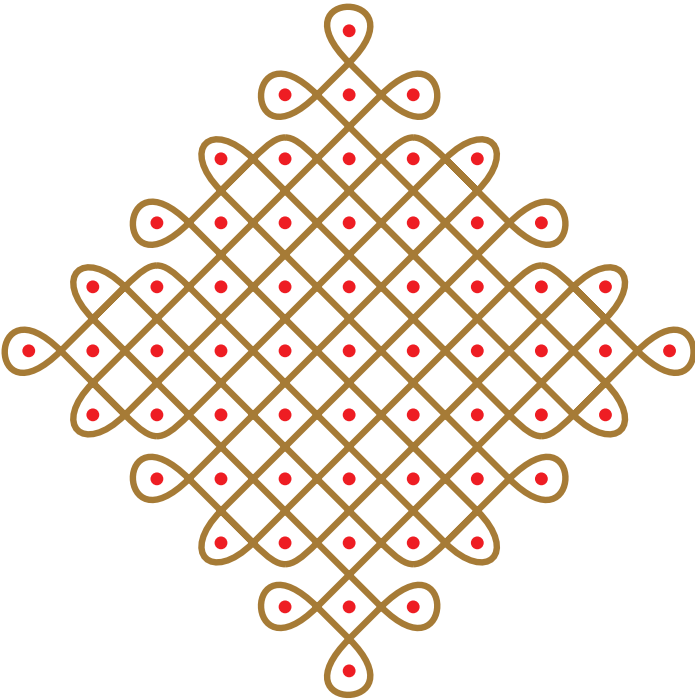
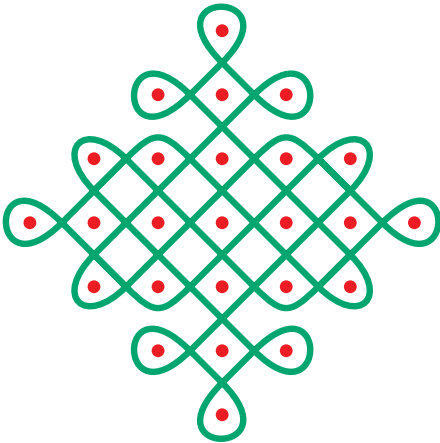


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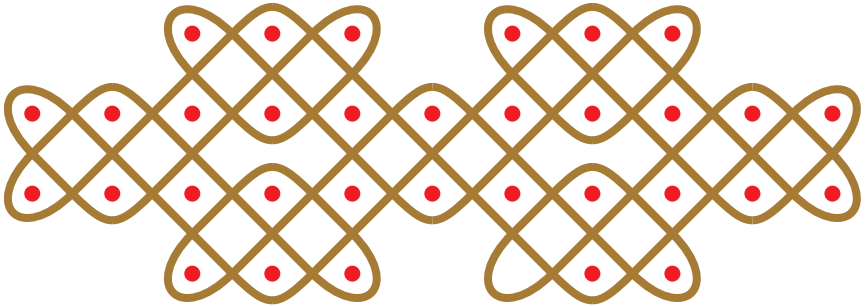
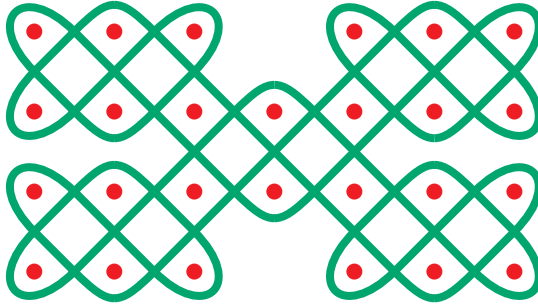
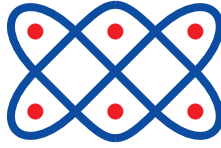
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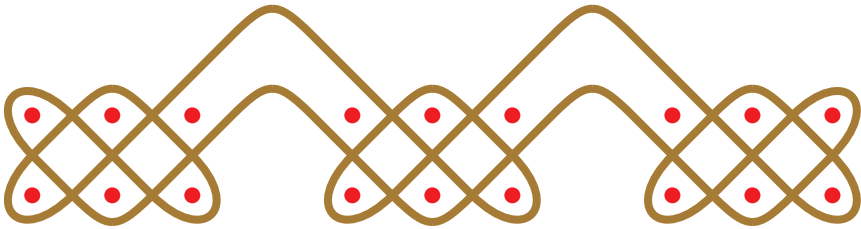
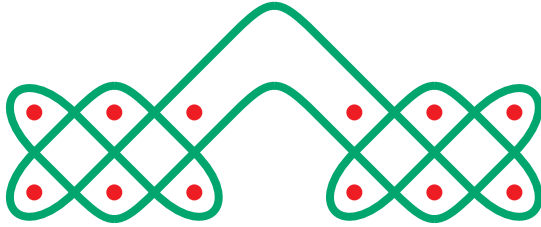


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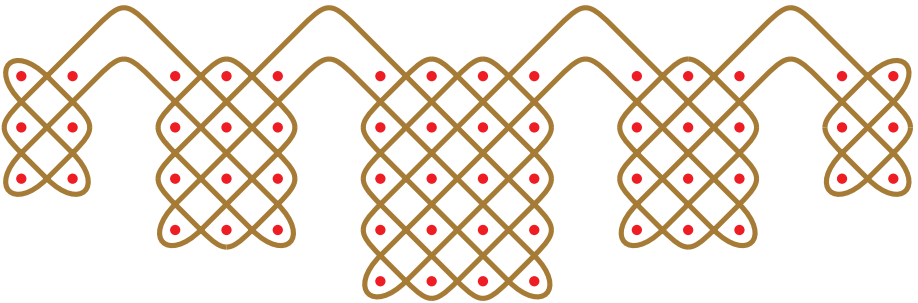
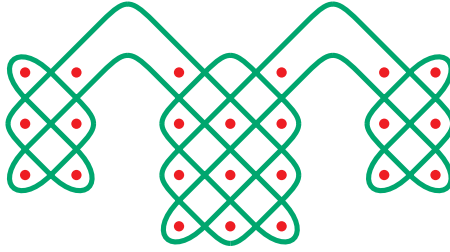


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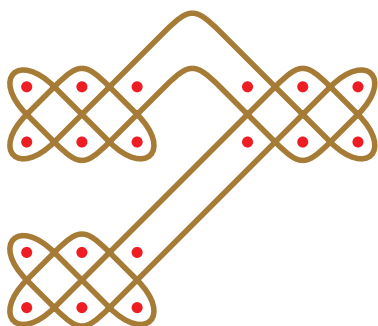
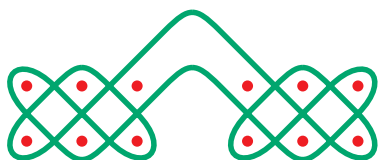


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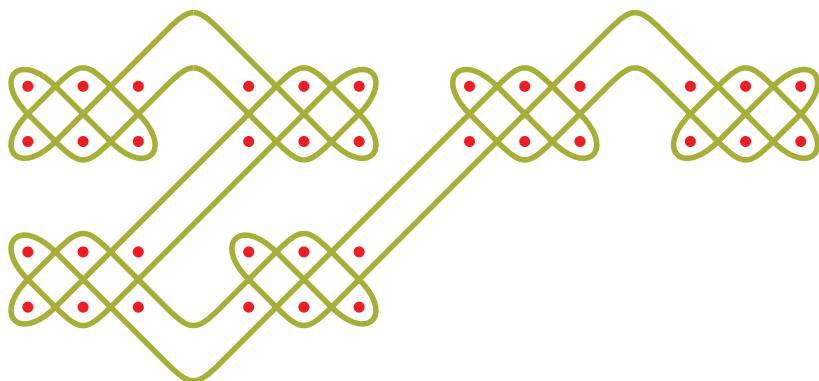
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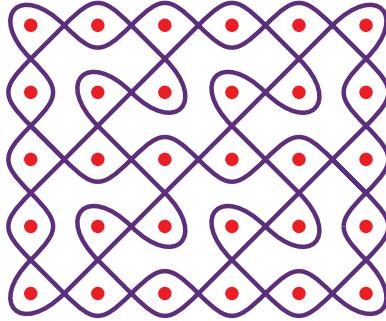


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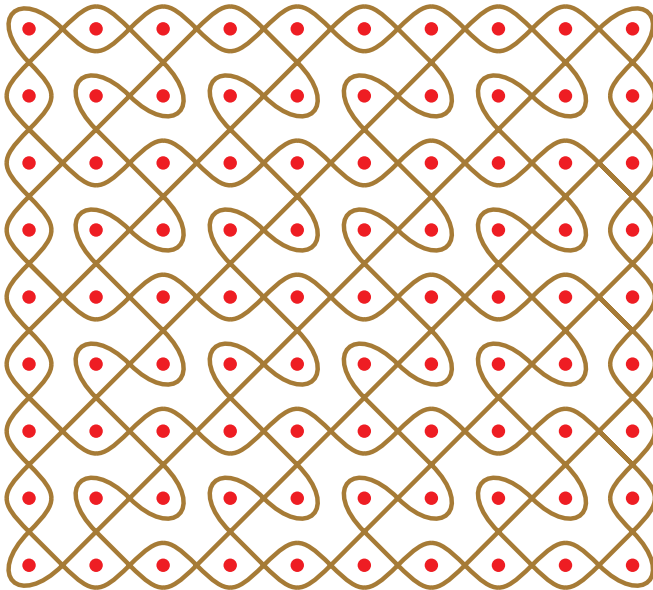
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Exemplos de SONA tradicionais

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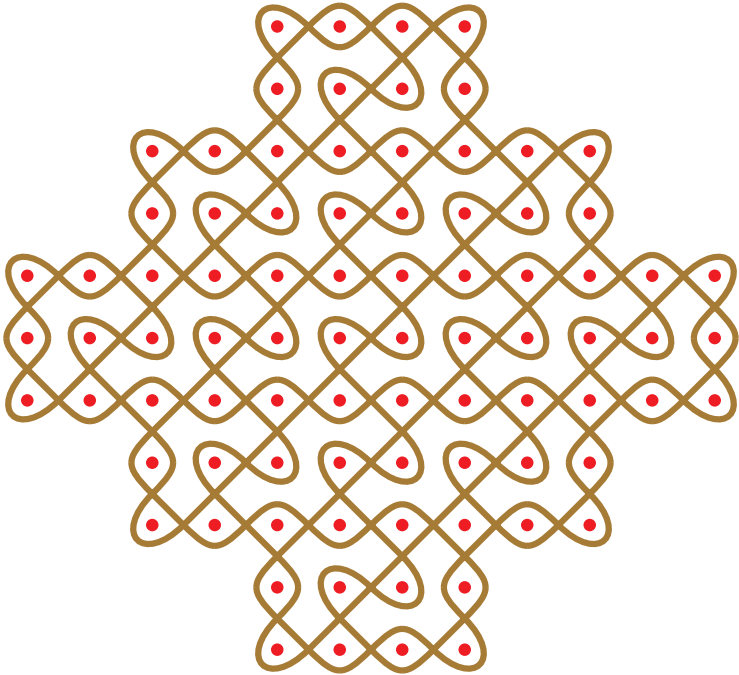
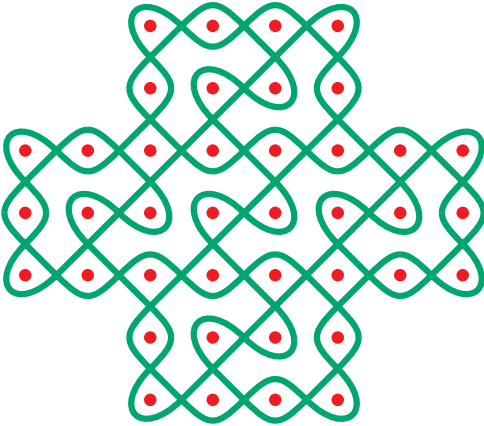


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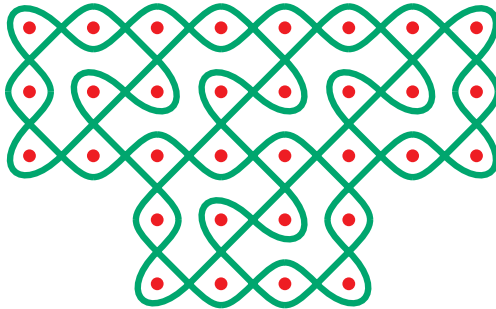
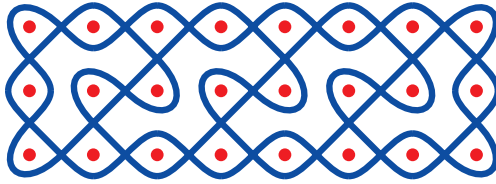
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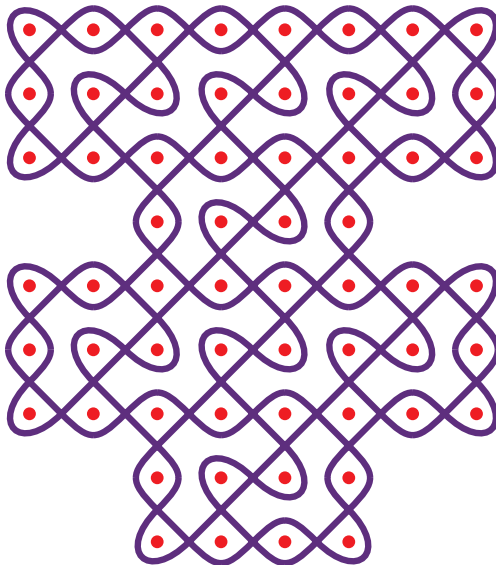


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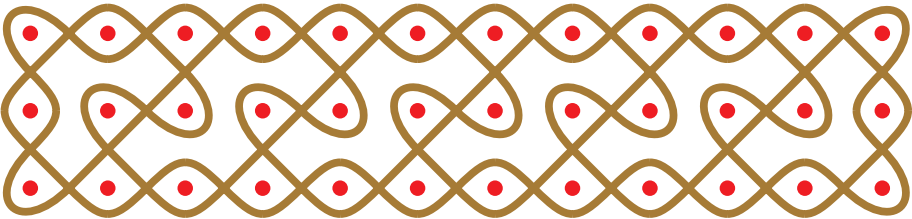
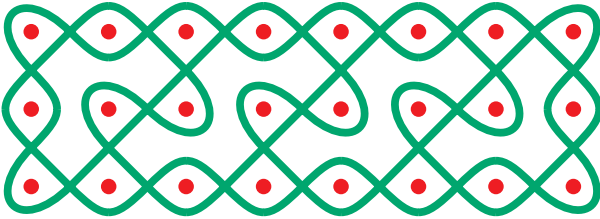
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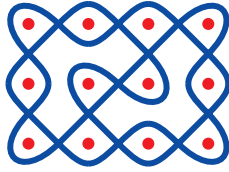
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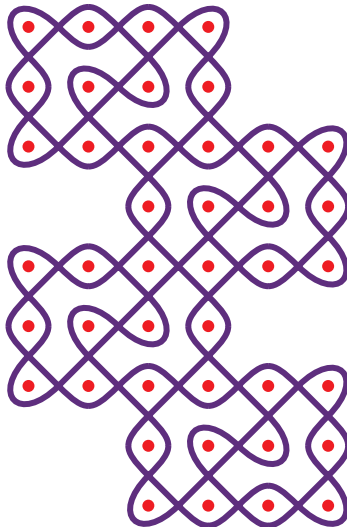
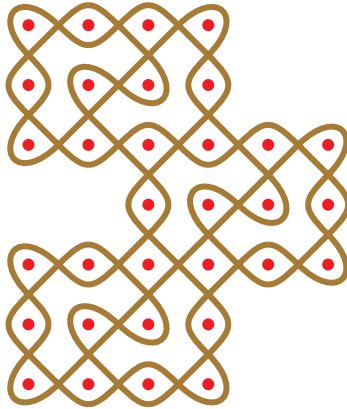


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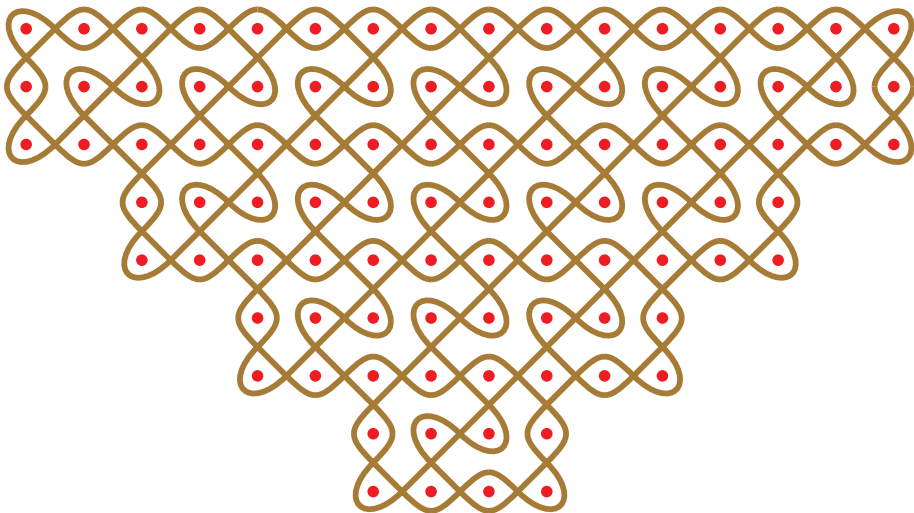
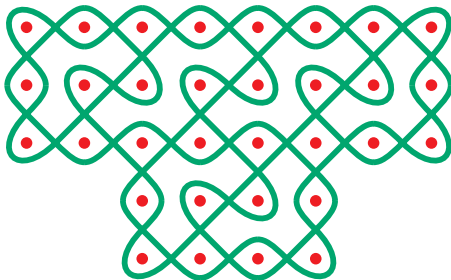
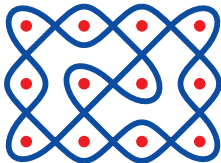
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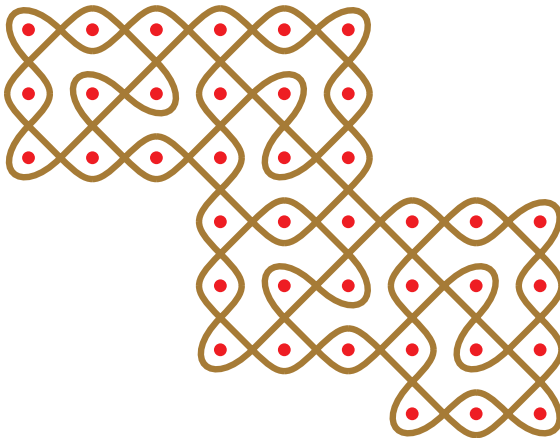
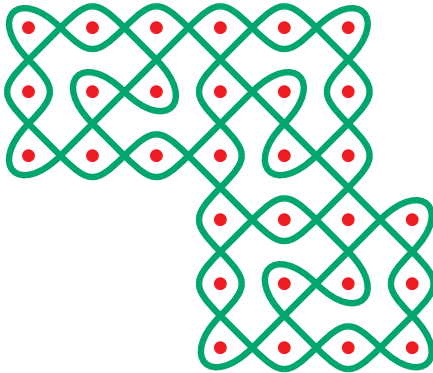
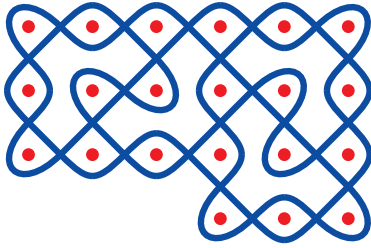
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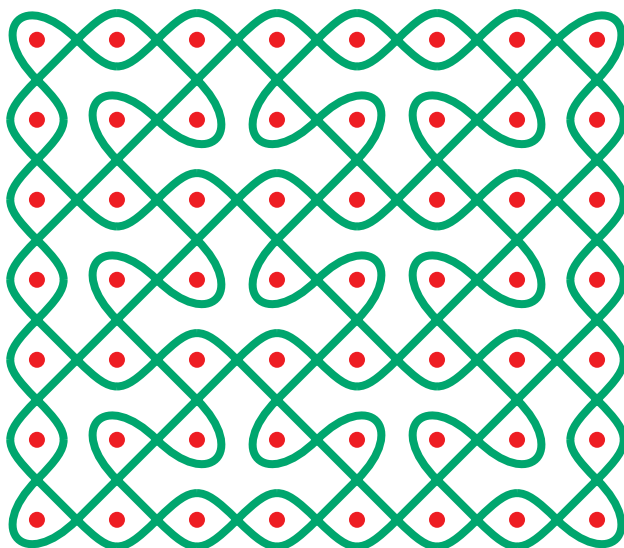
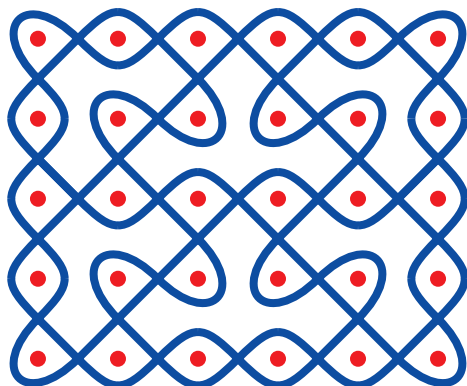


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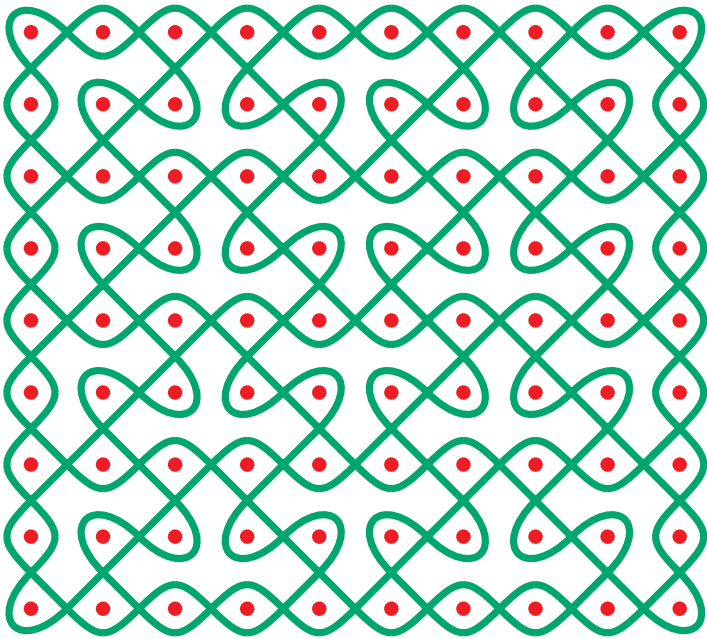
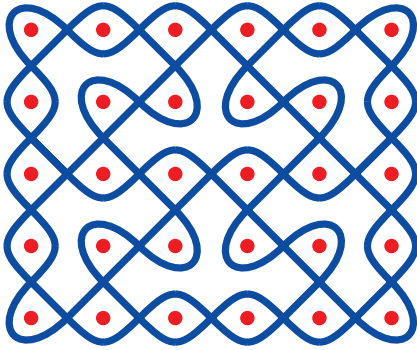
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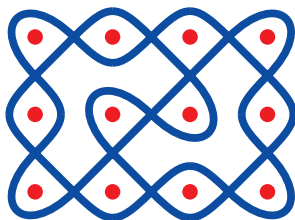
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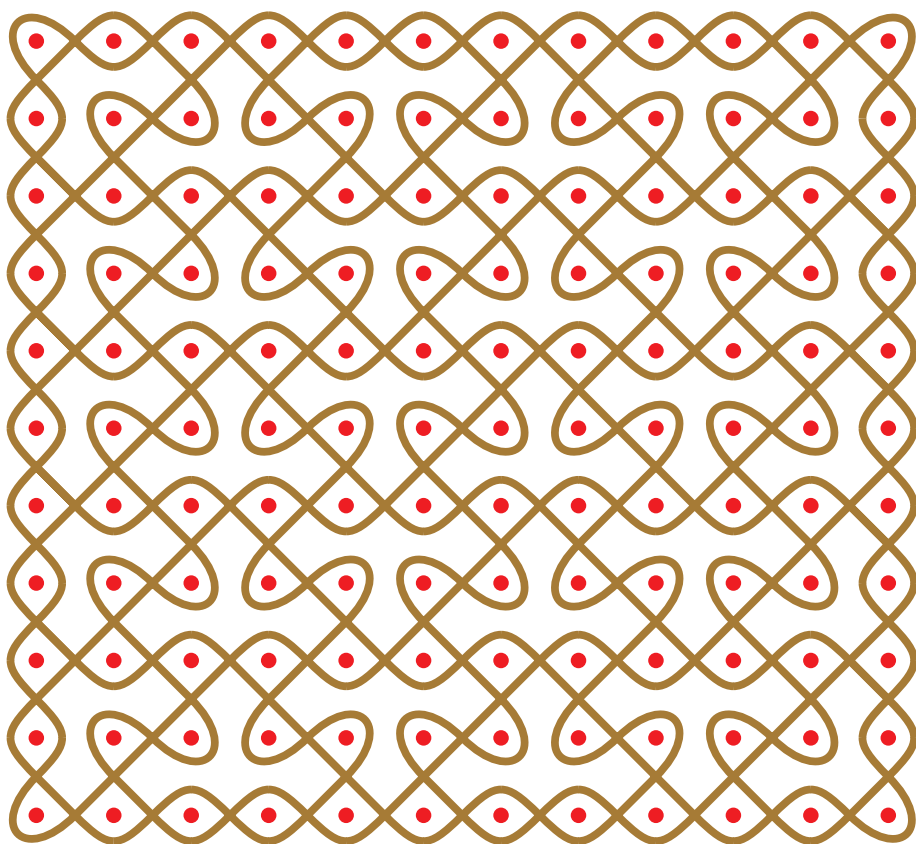


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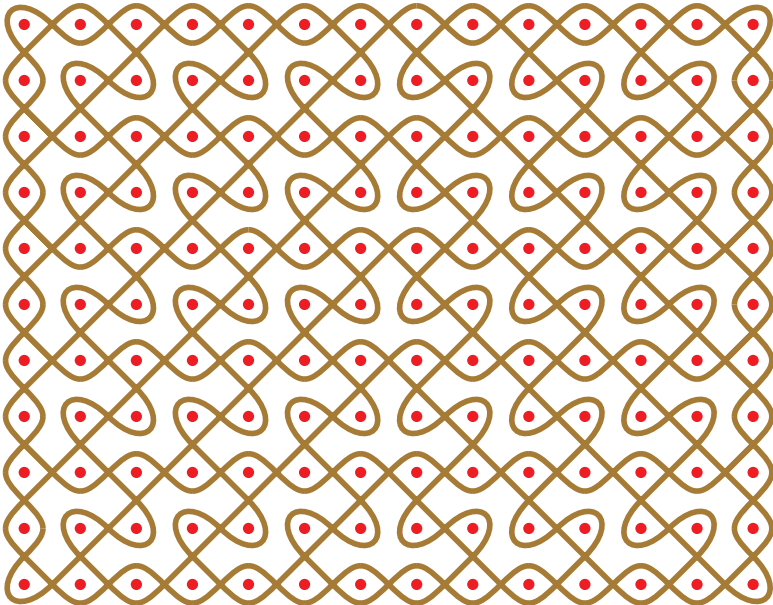
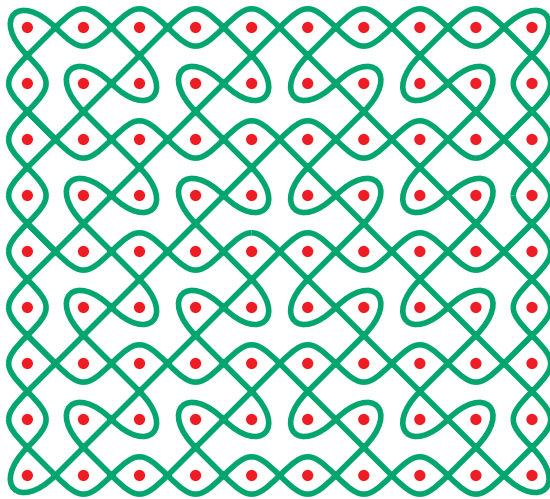


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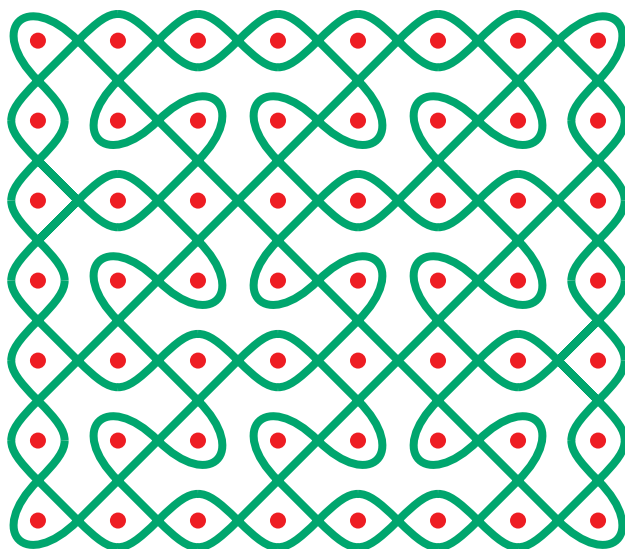
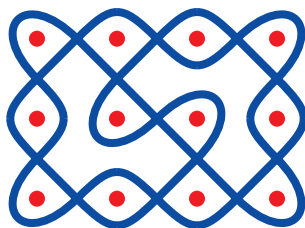


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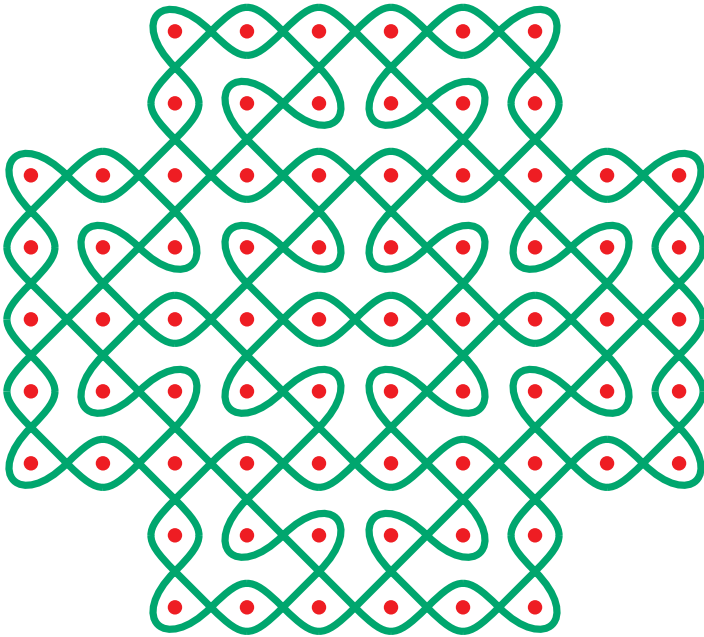
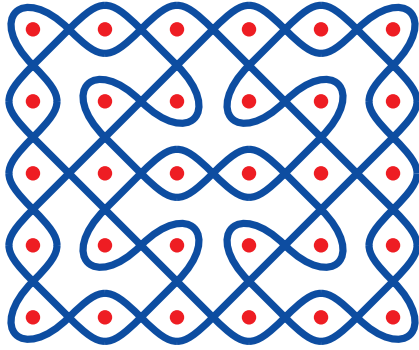
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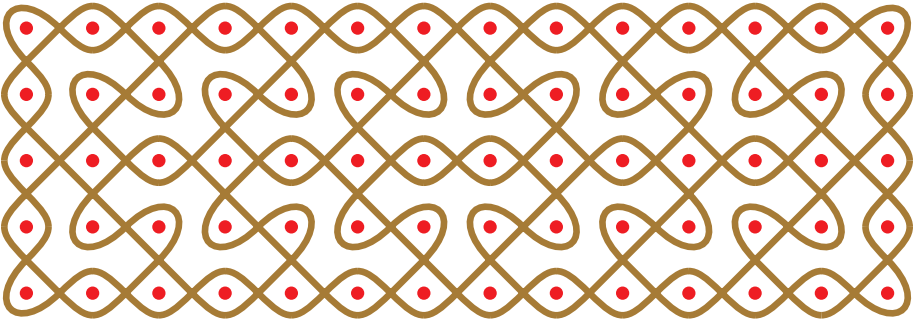
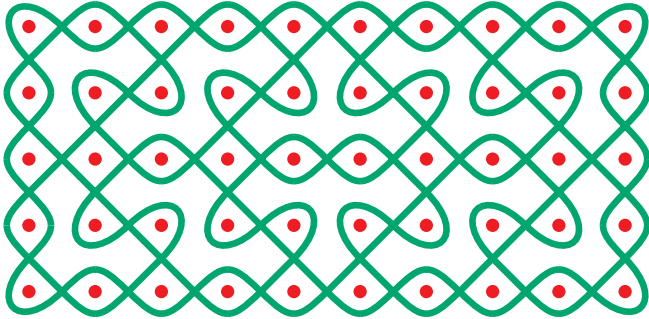


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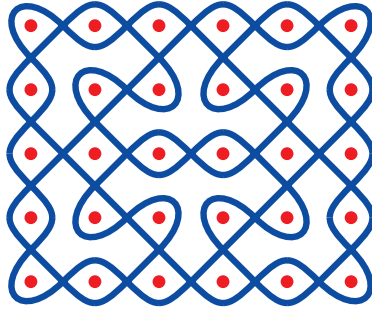
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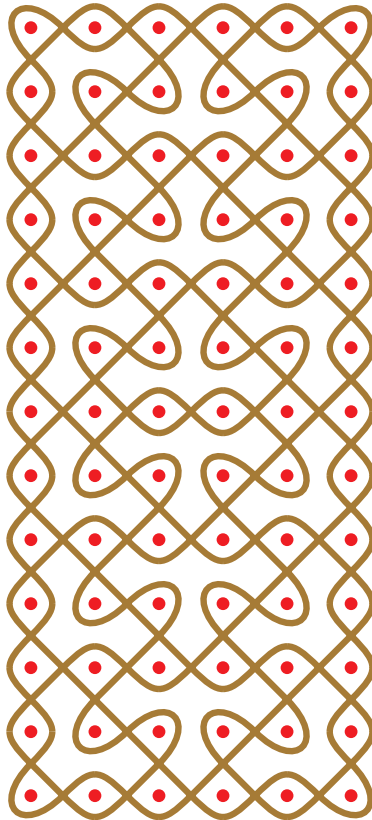
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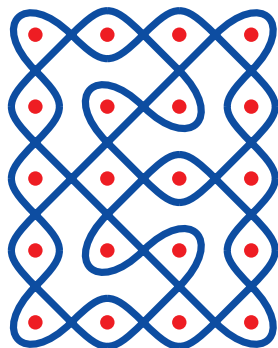
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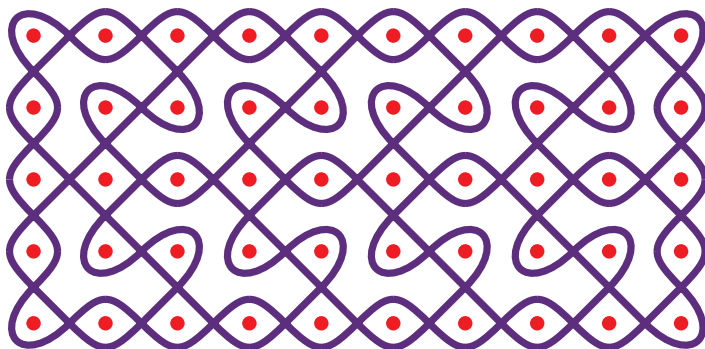
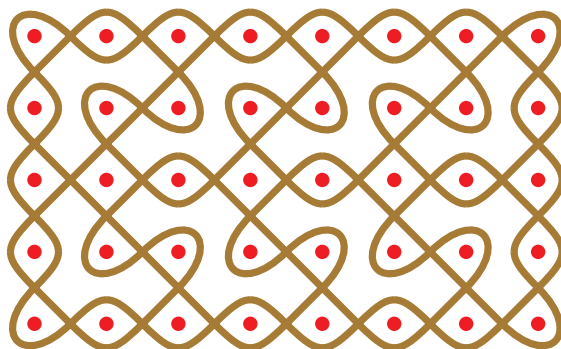
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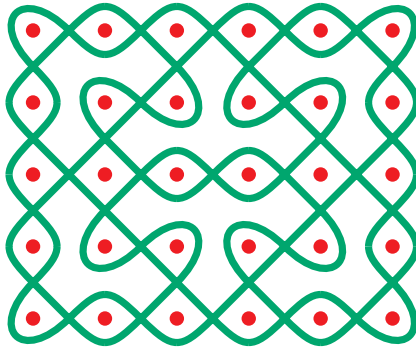


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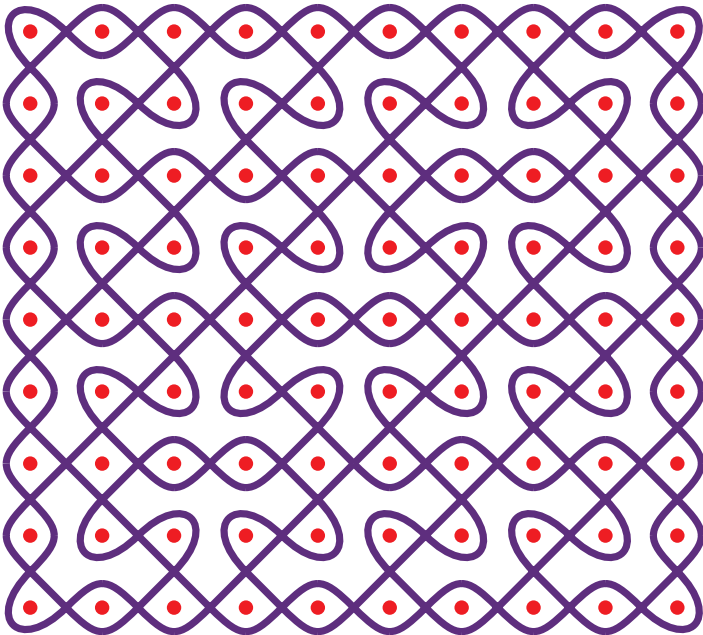


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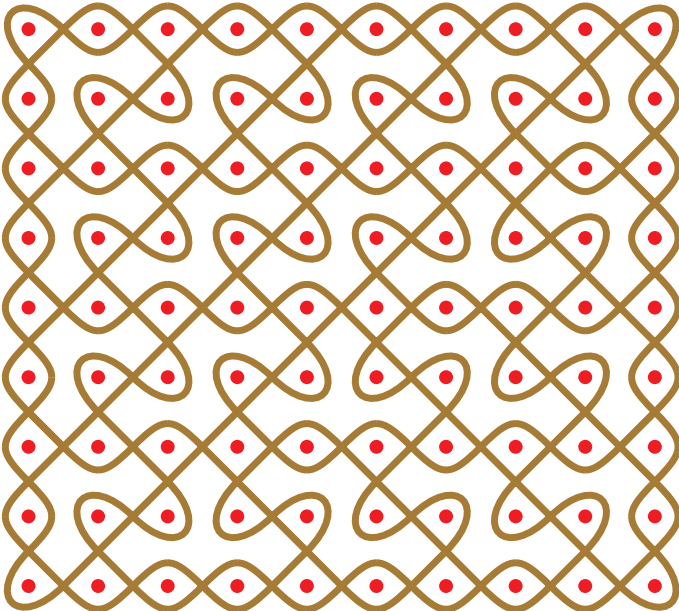
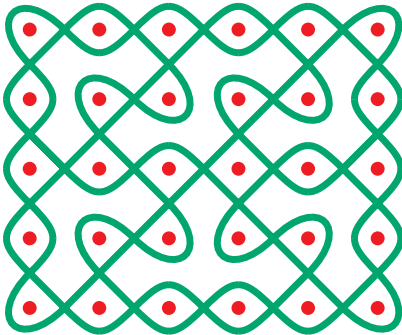


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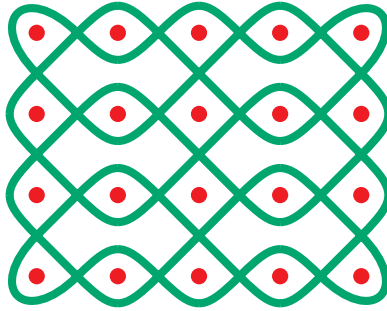
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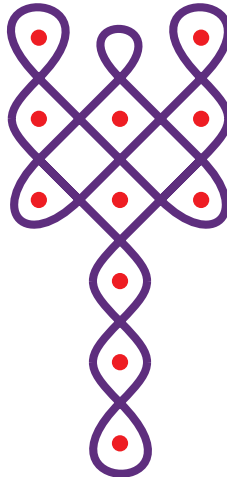
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Exemplos de SONA tradicionais

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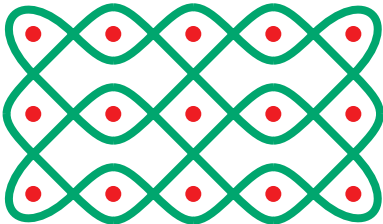
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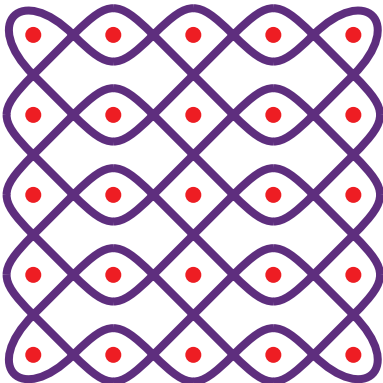
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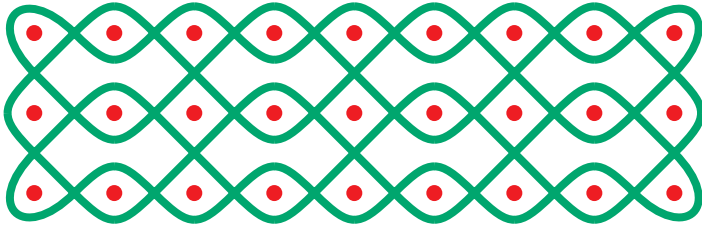
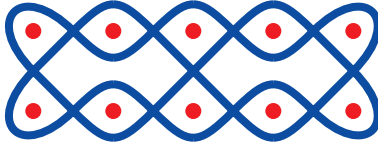


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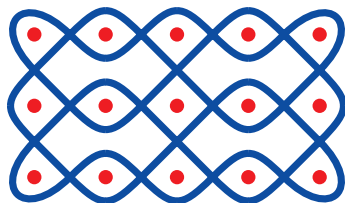
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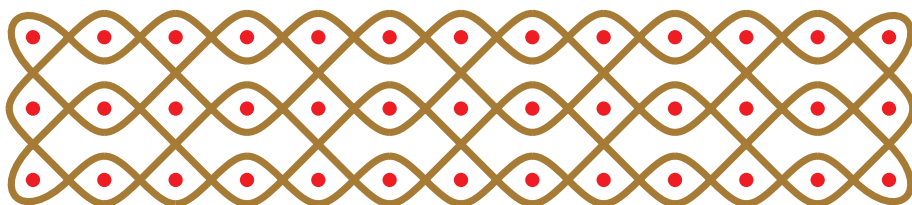
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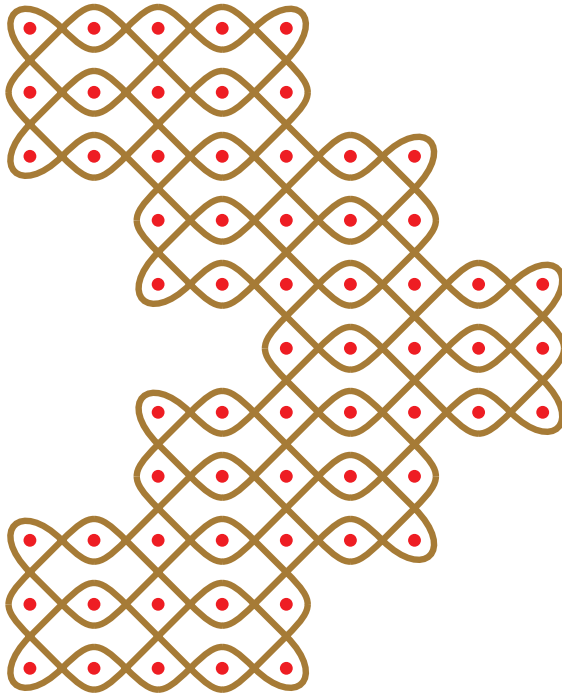
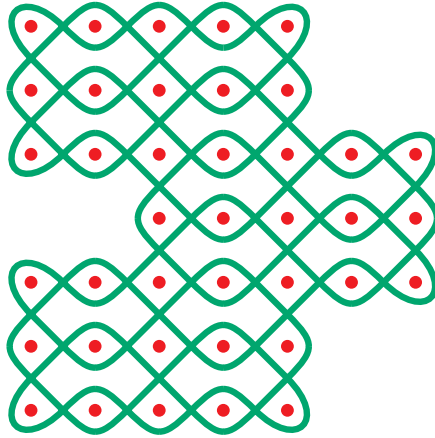


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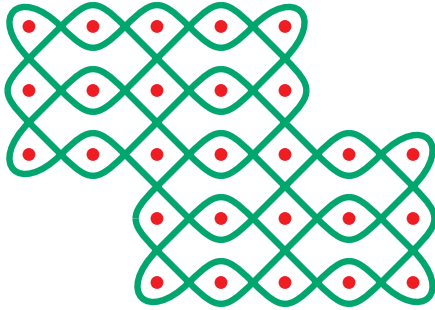
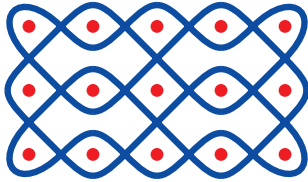


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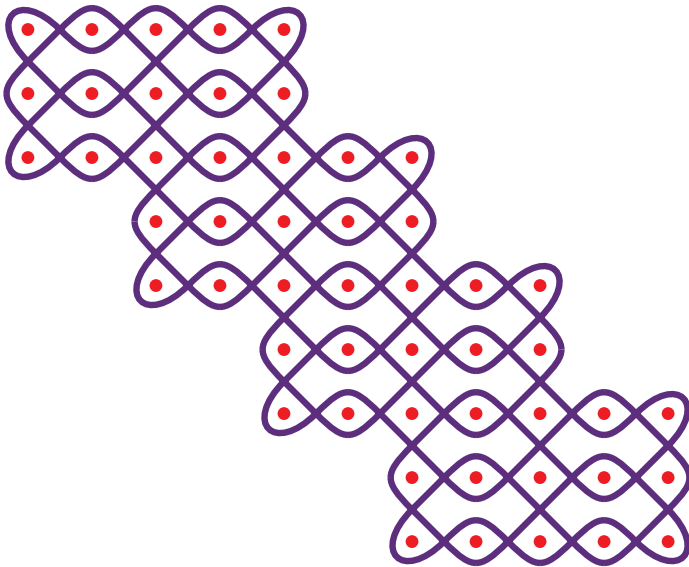
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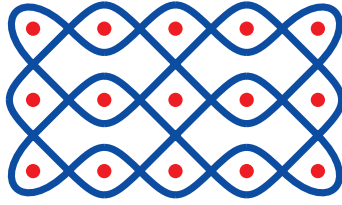
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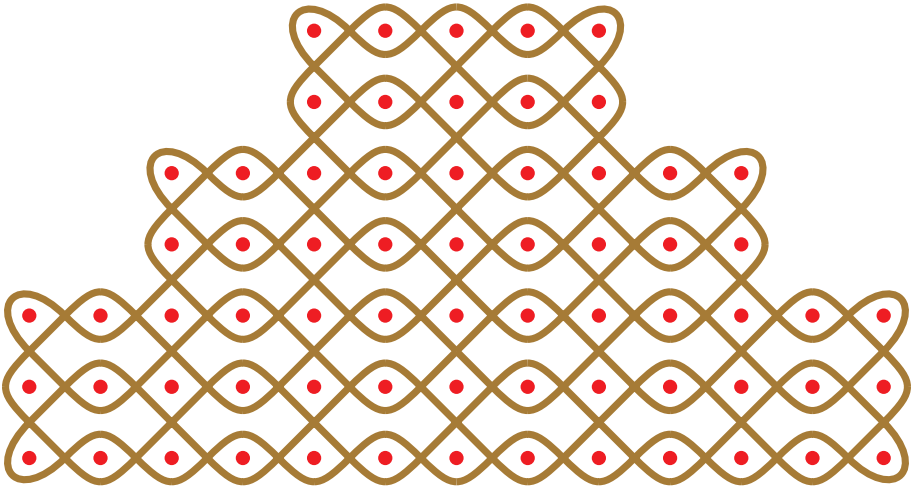
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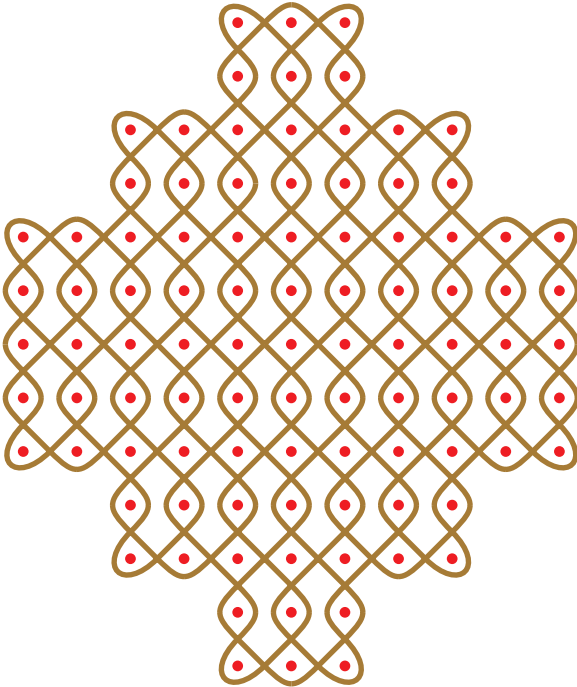
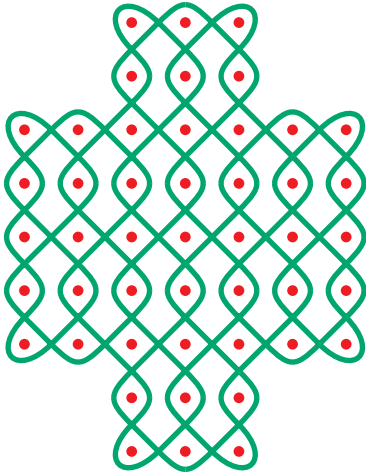
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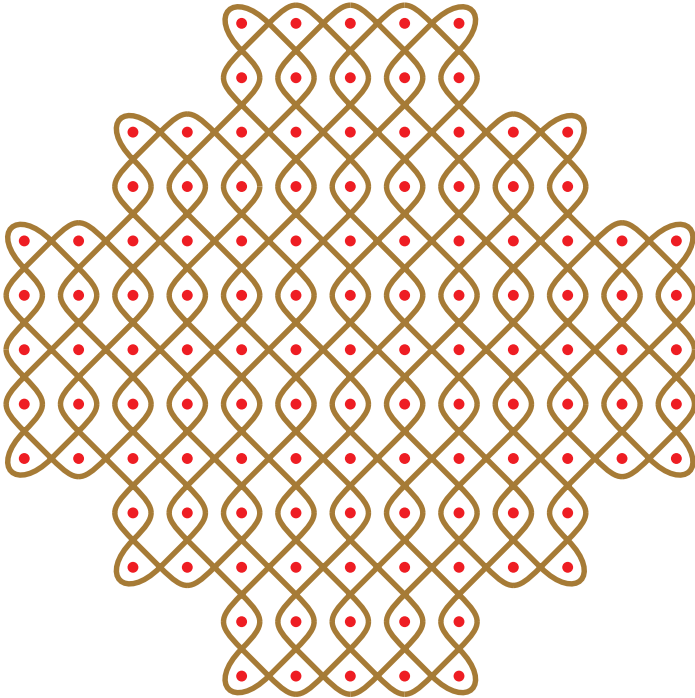
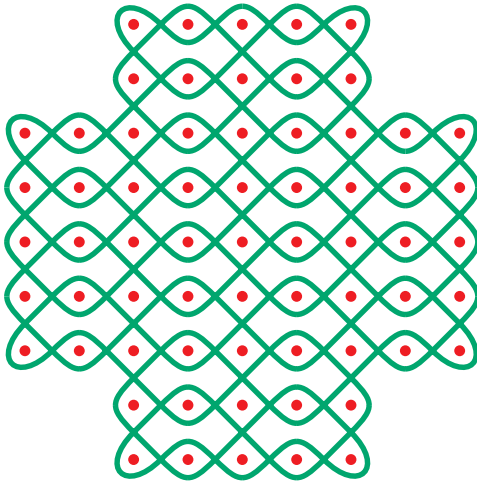
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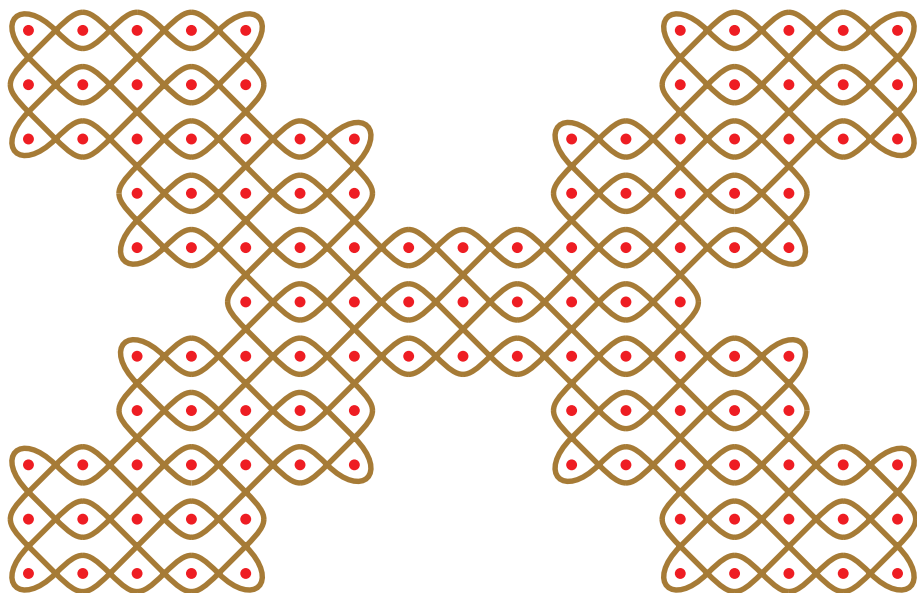
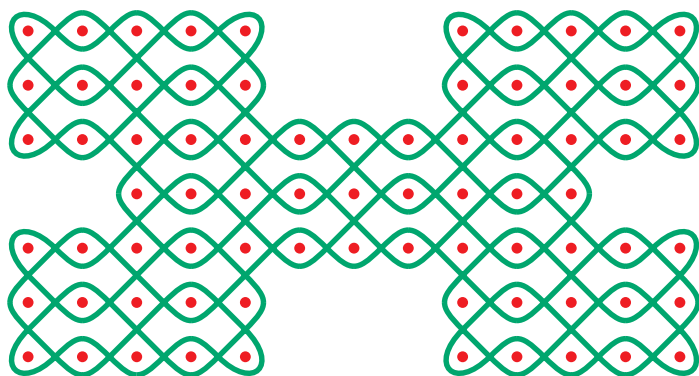
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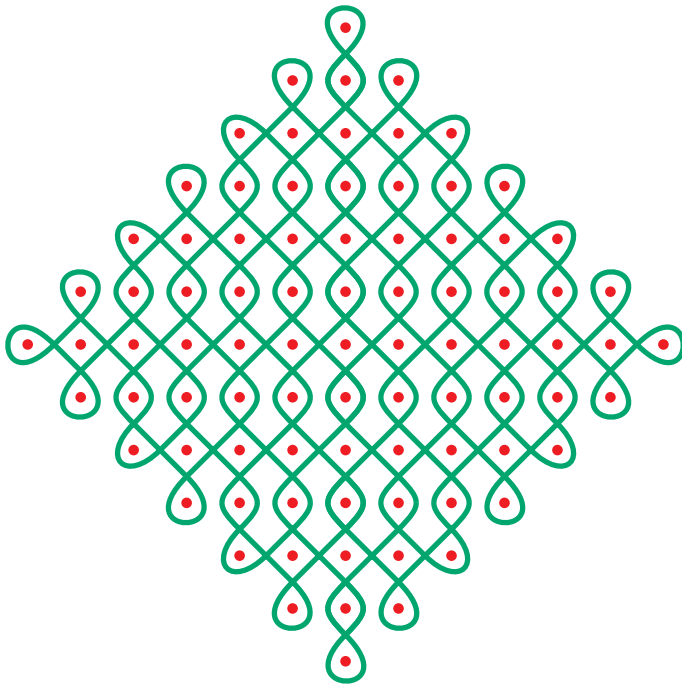
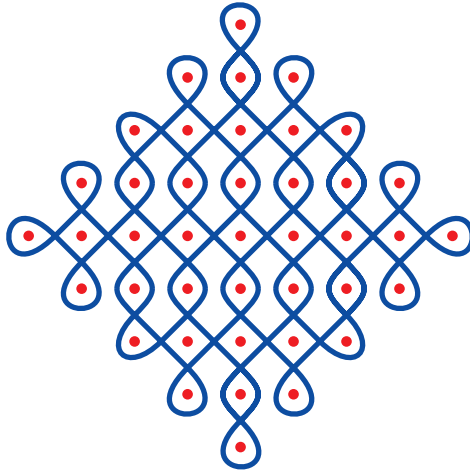


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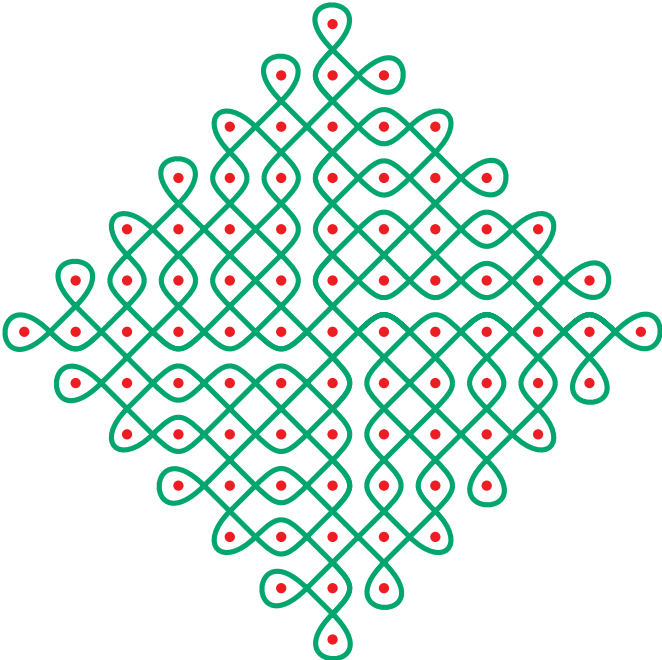
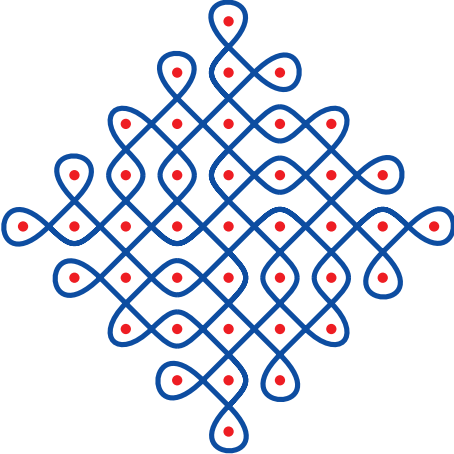
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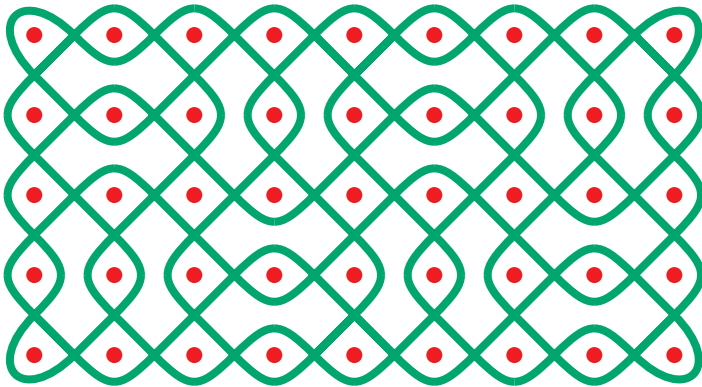
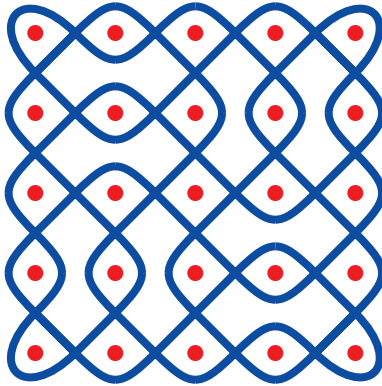
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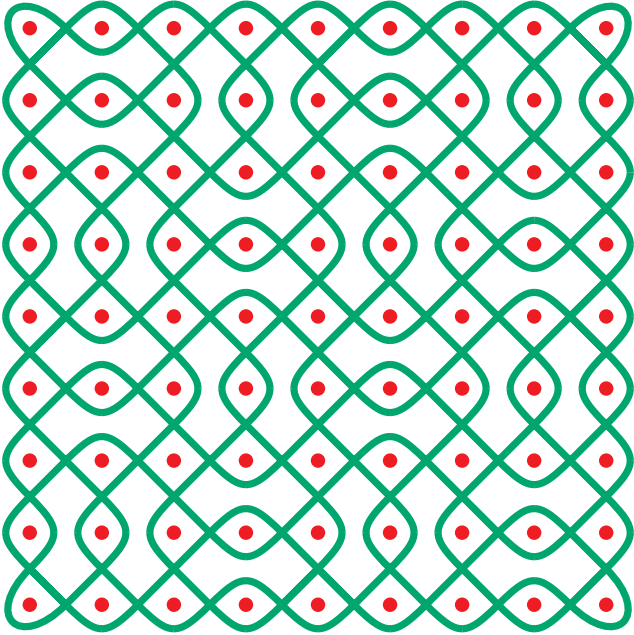
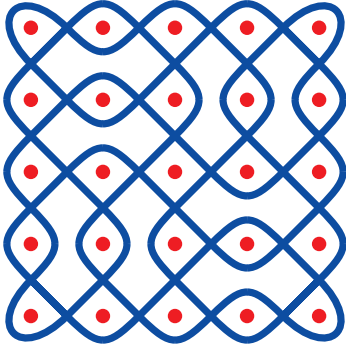


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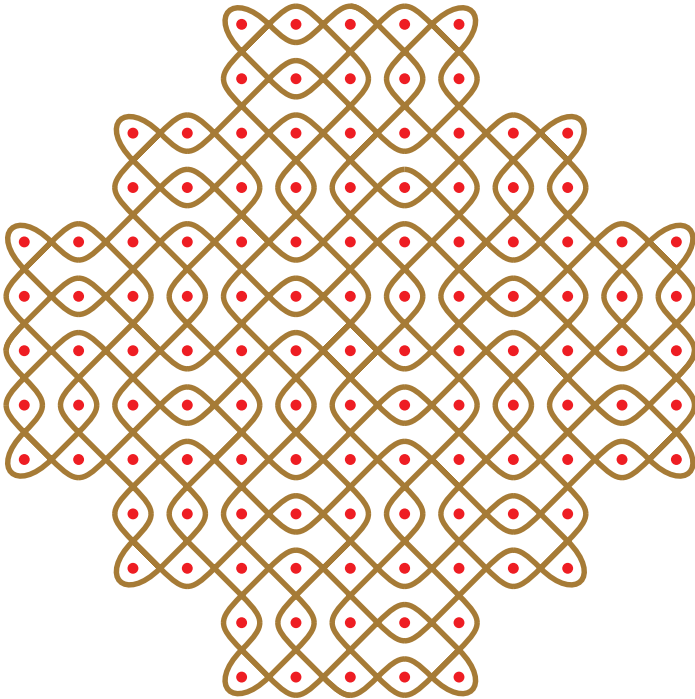
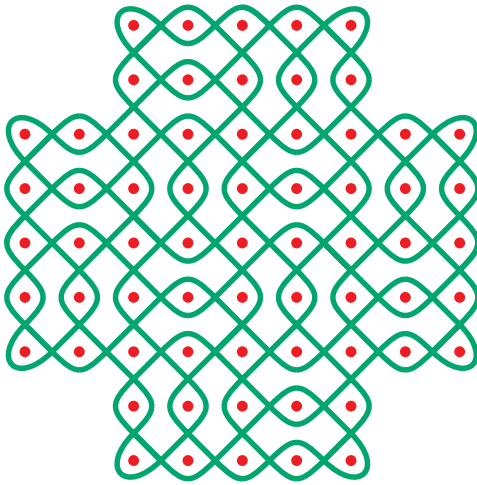
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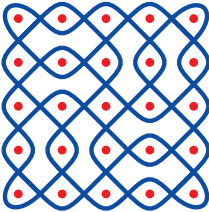
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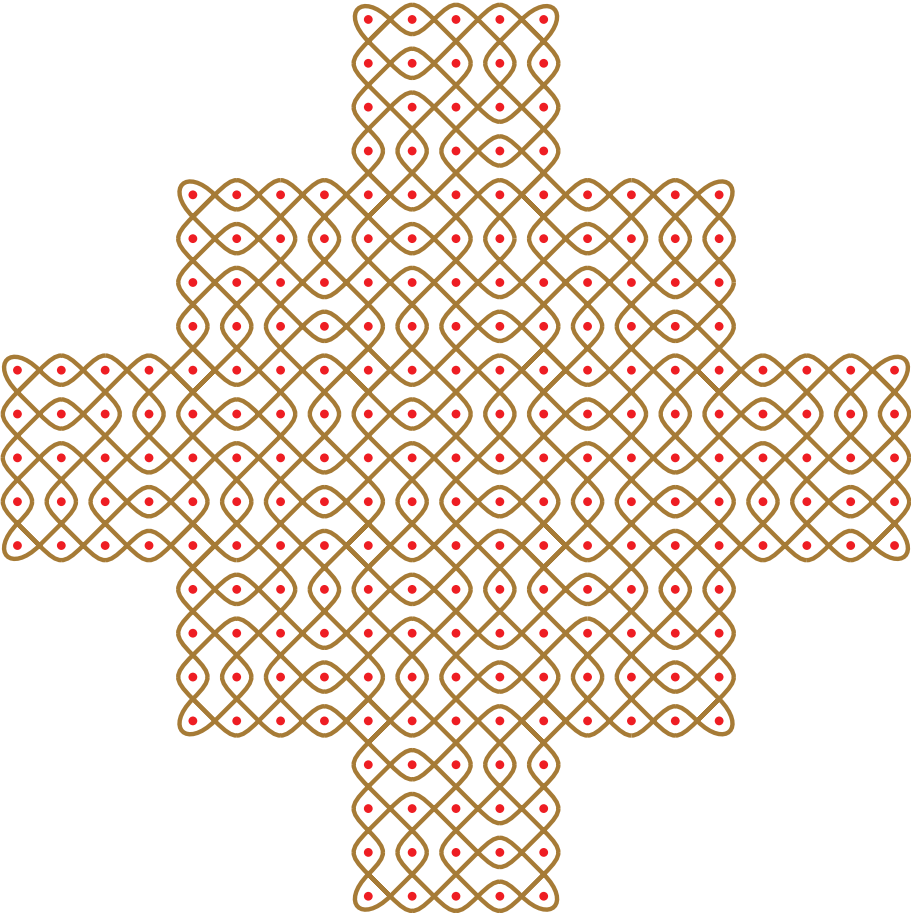
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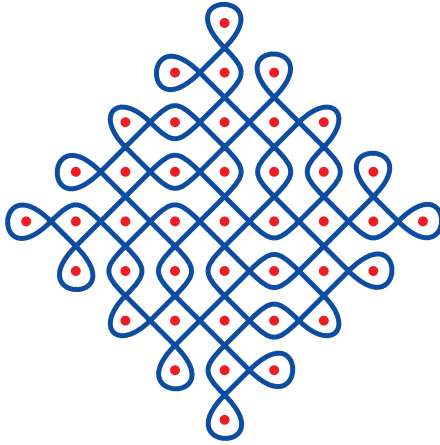


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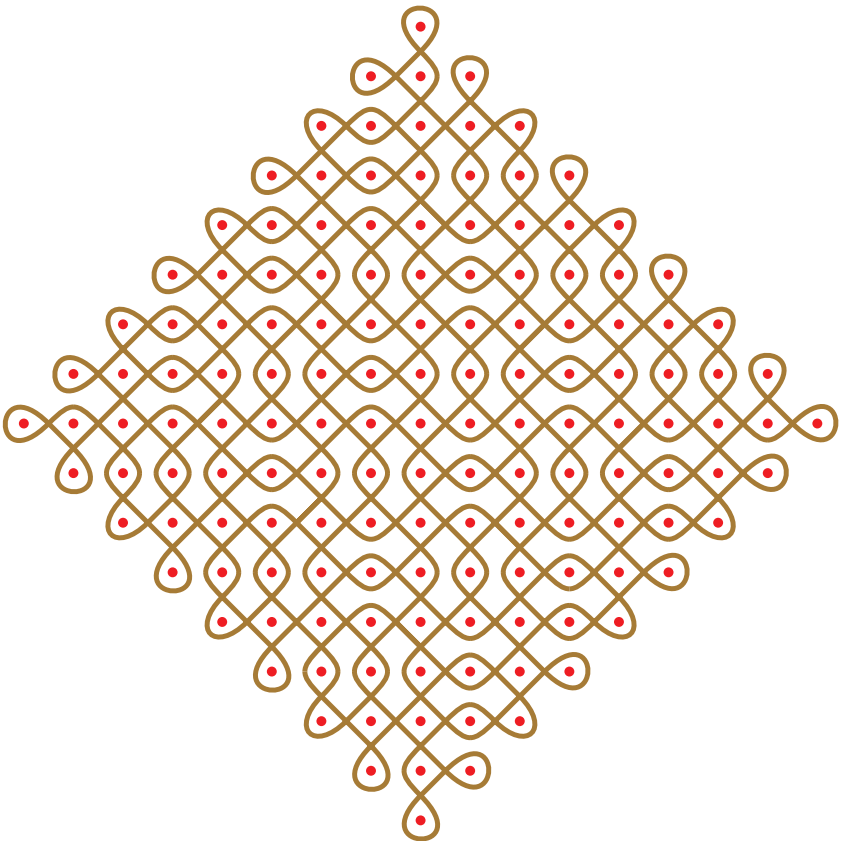


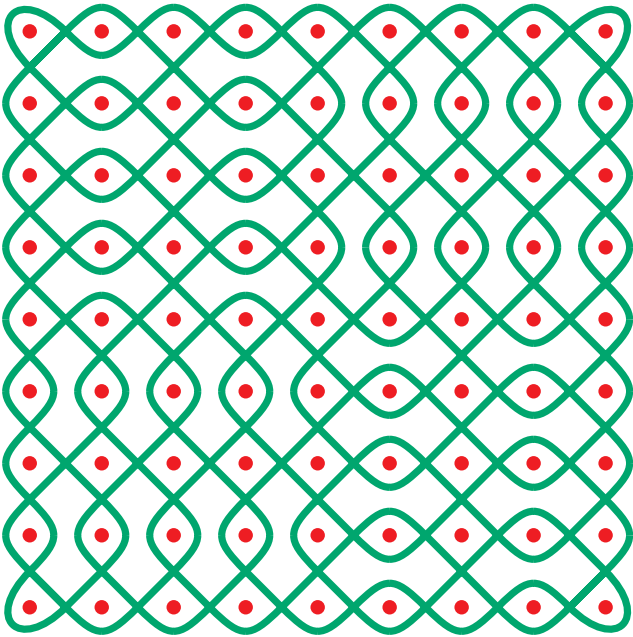
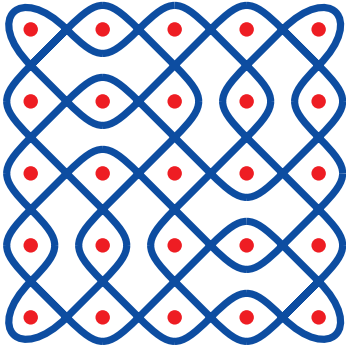
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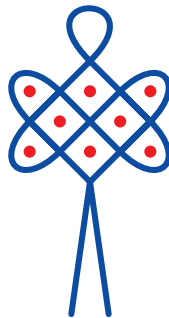
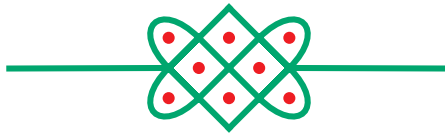
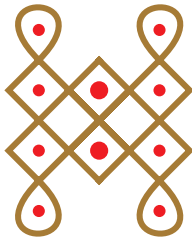
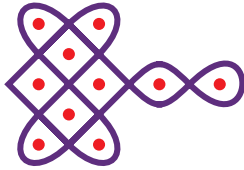


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Exemplos de SONA tradicionais

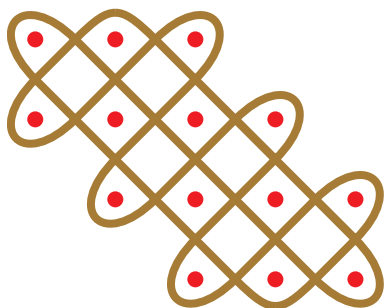
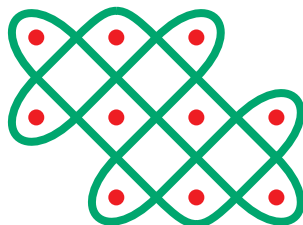
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insecto "misepe"

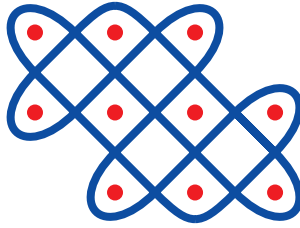
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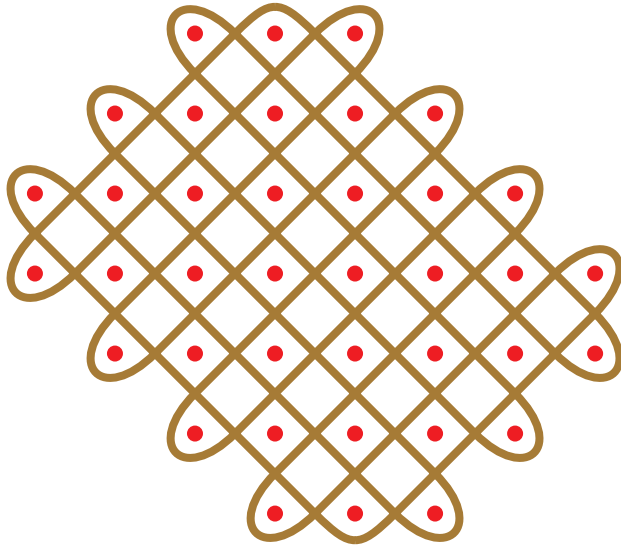


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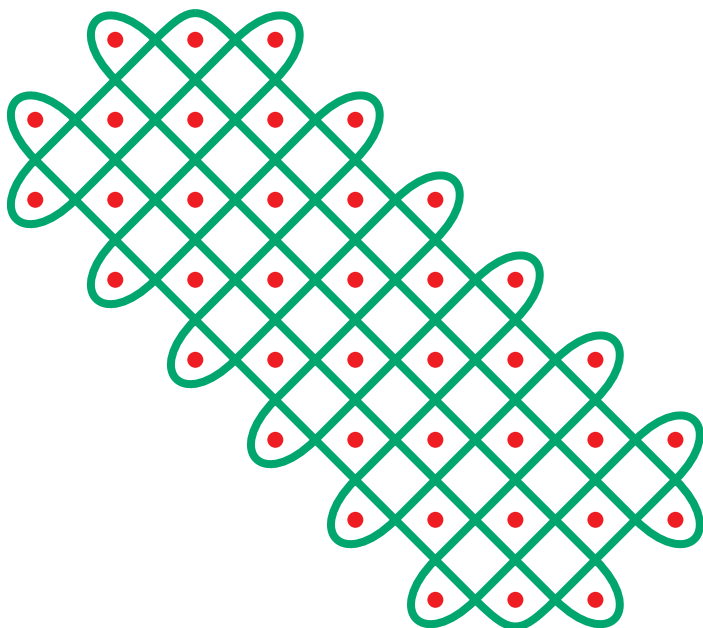
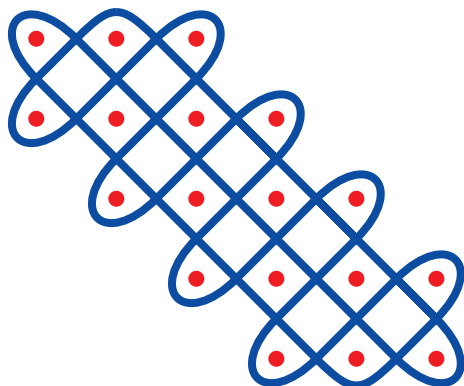


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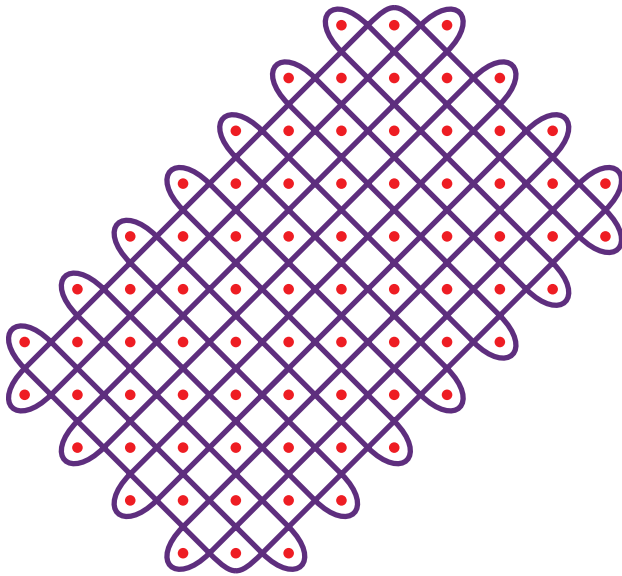
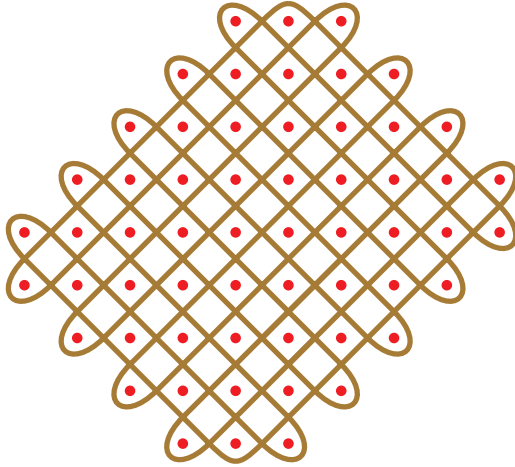
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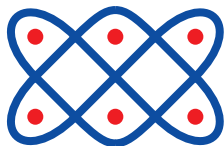
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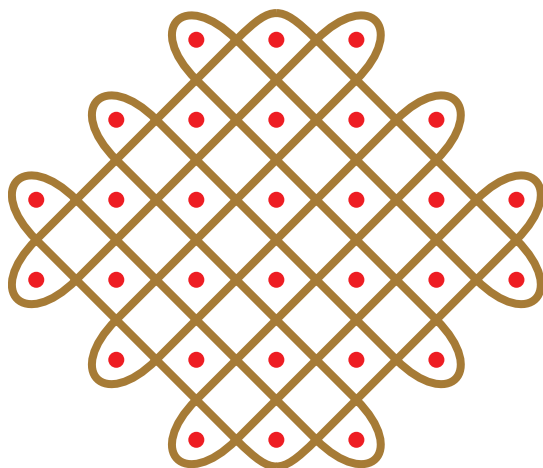


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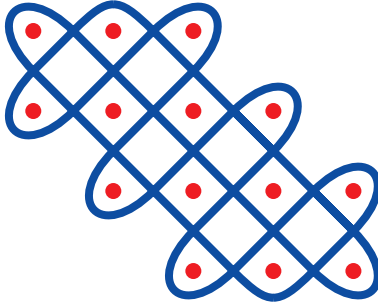


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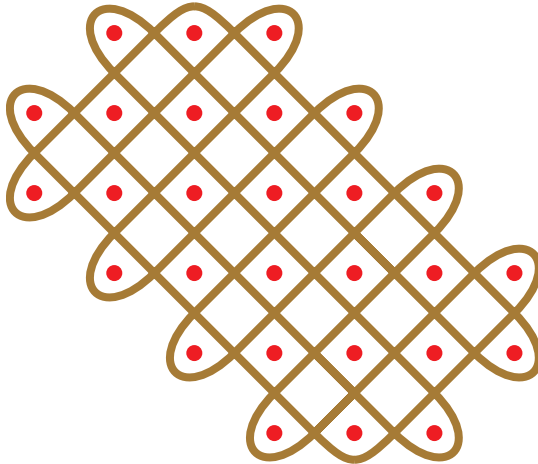


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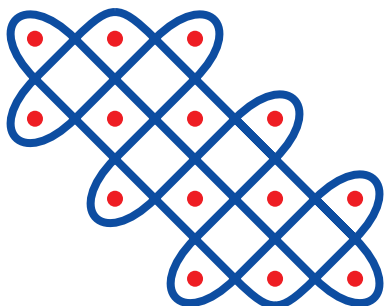
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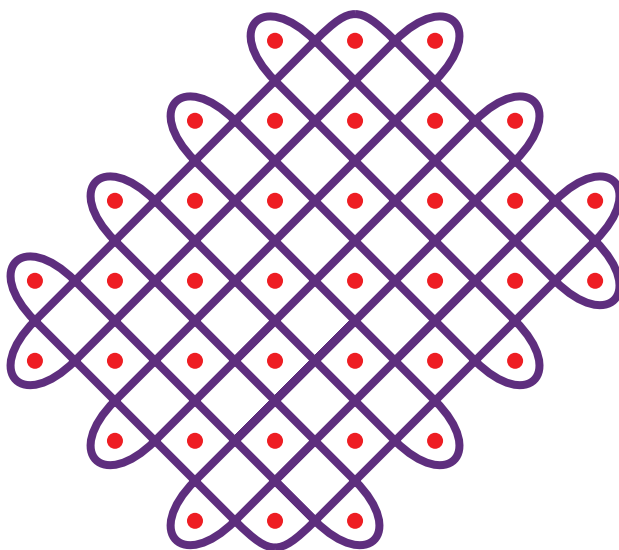
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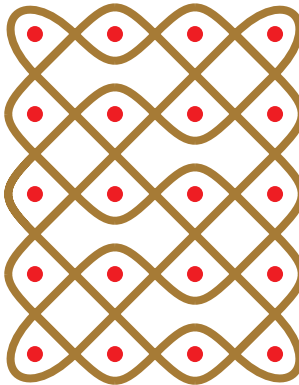
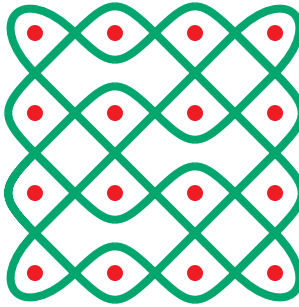
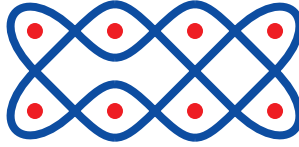


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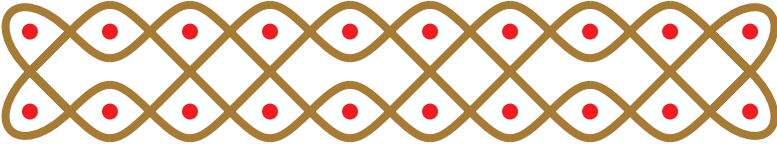
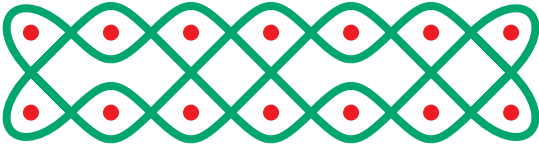


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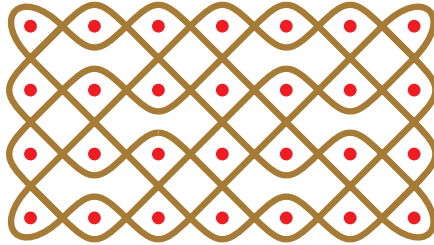
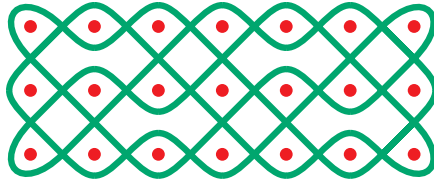
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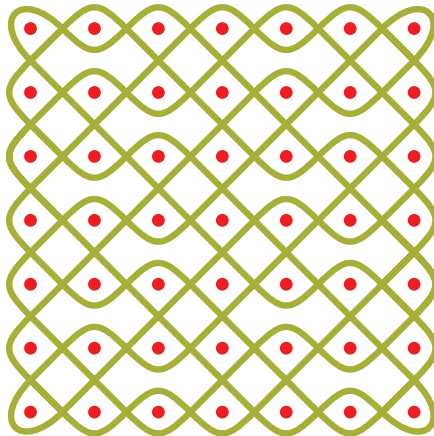
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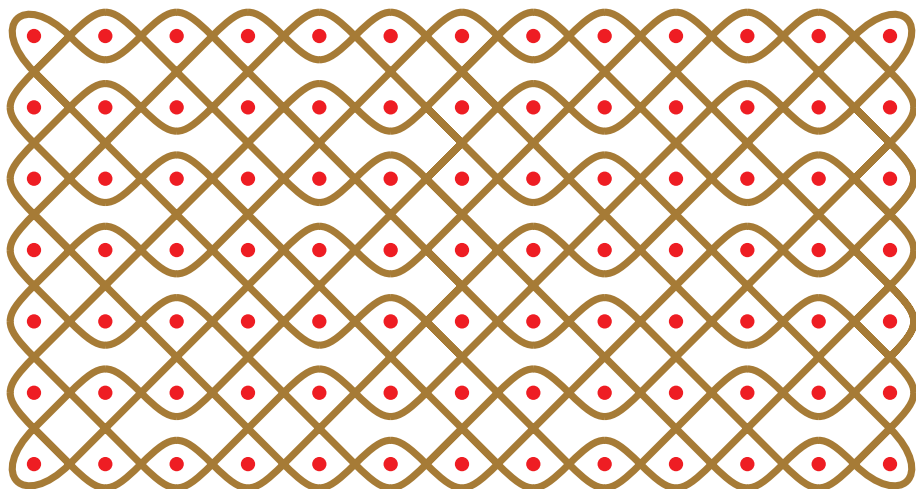
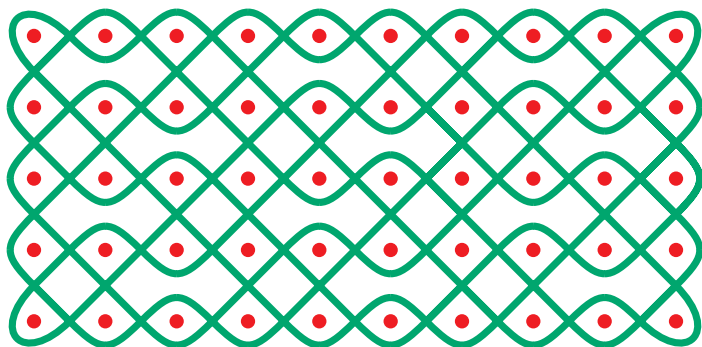
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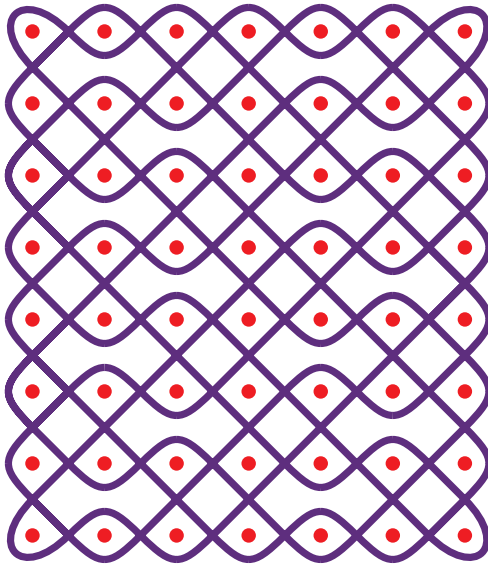
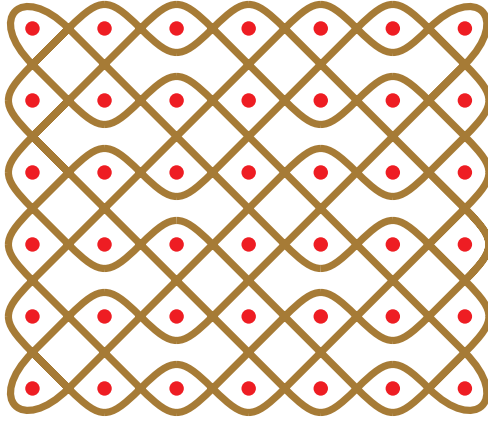
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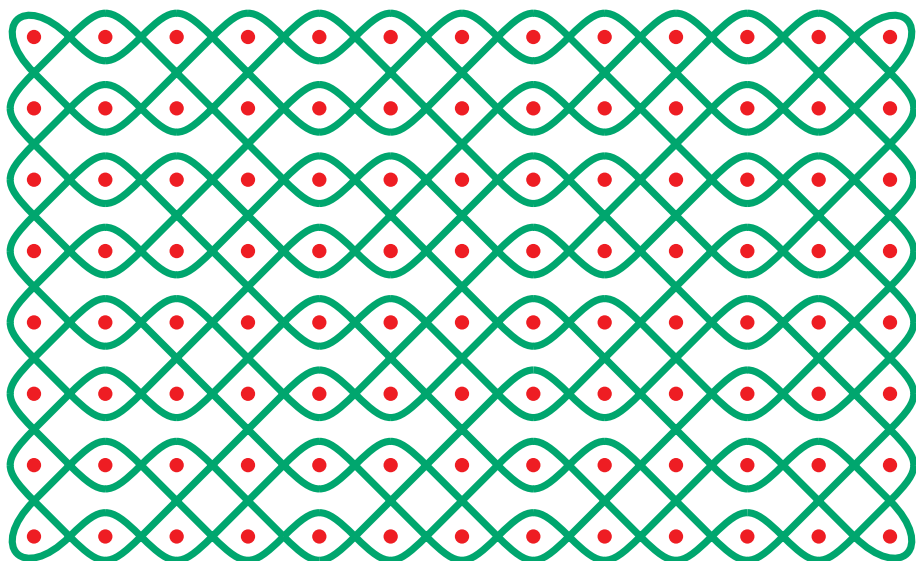
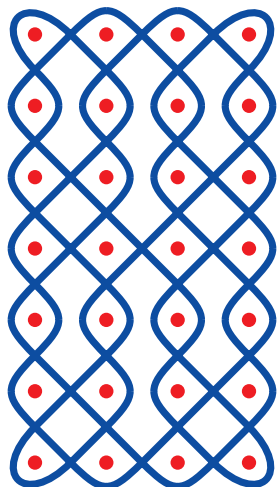
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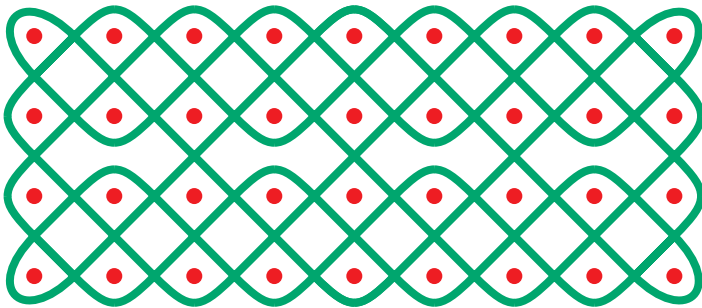
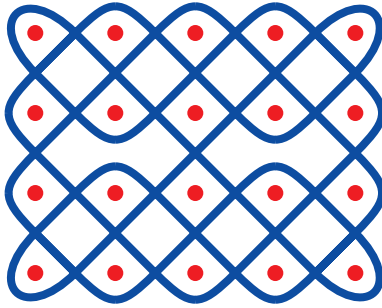


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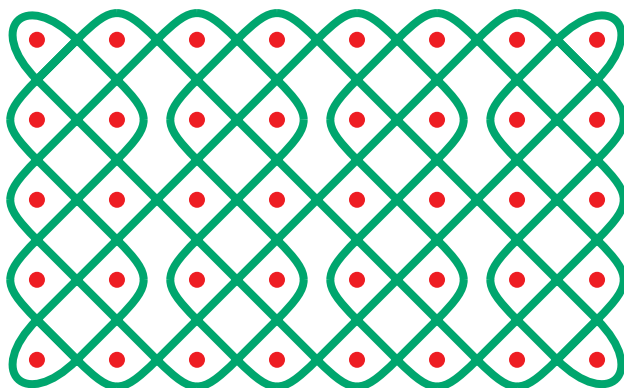
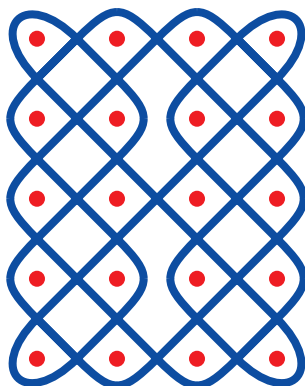
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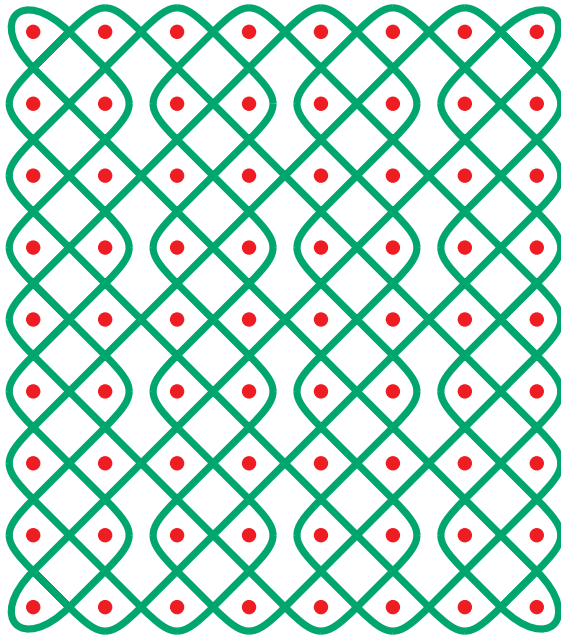
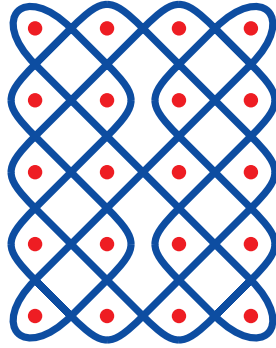
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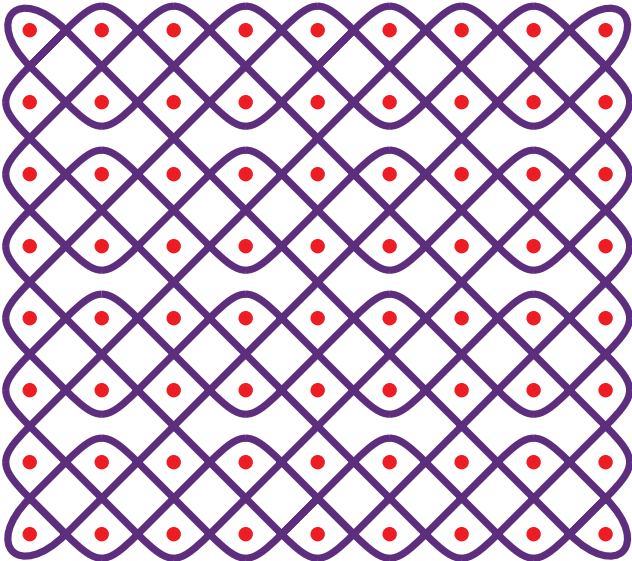
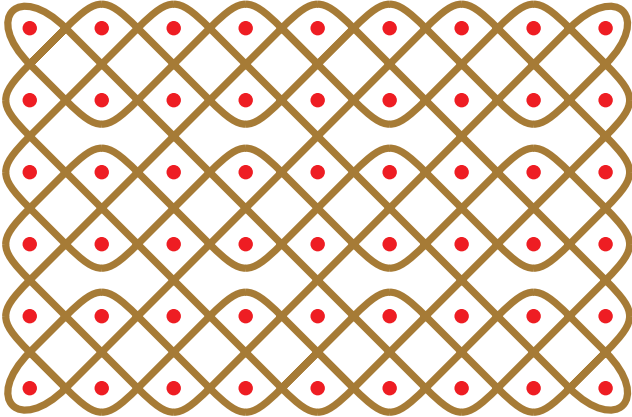


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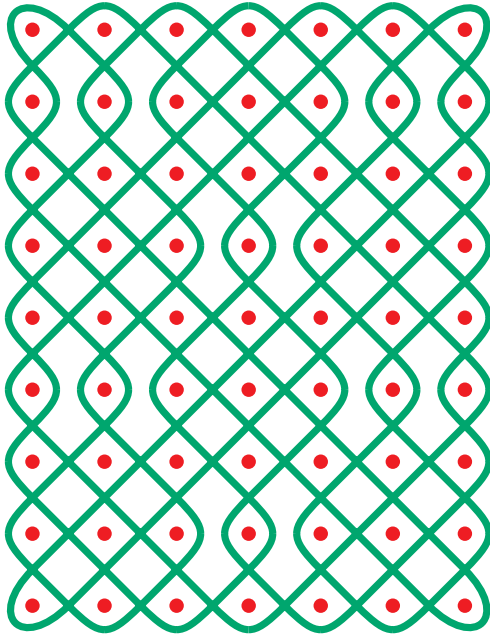
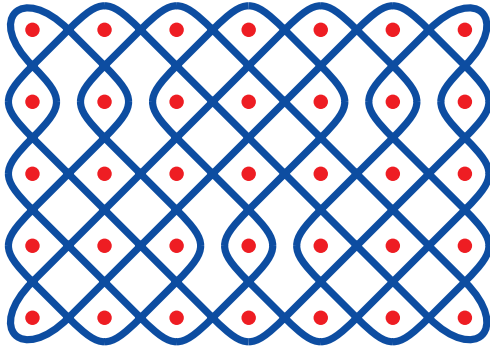
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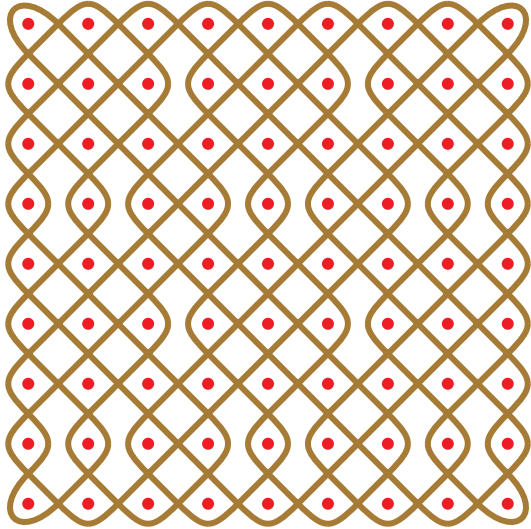
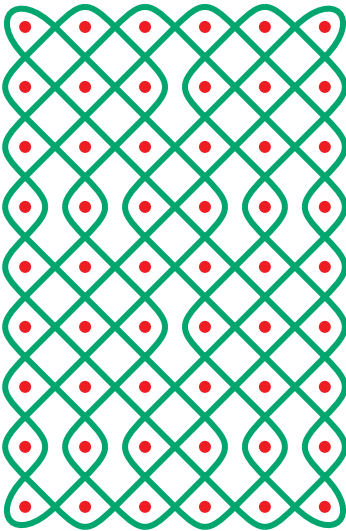


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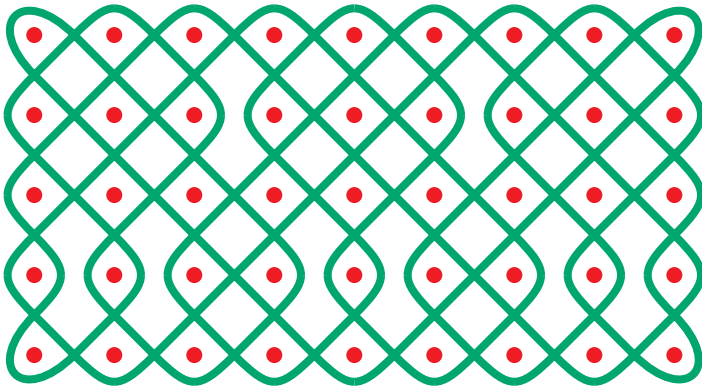
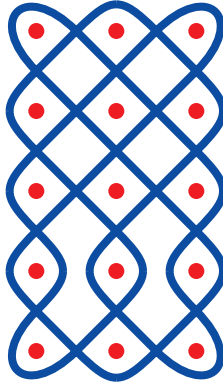
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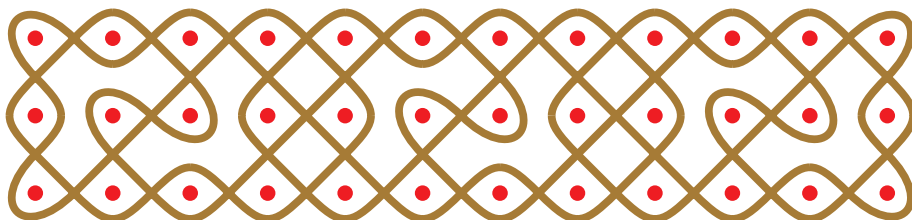
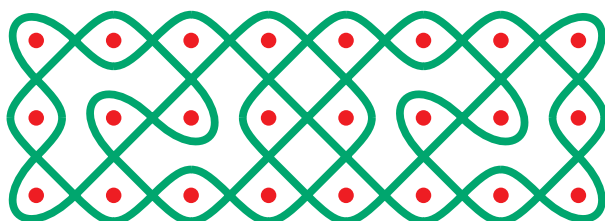
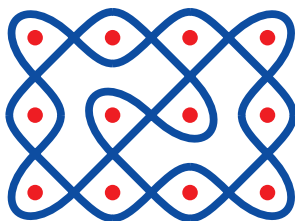


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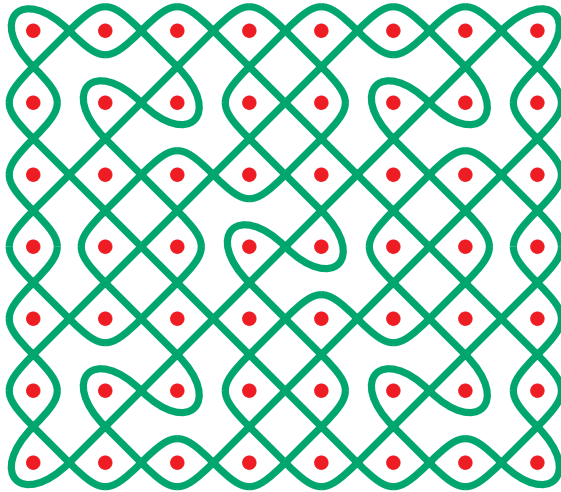
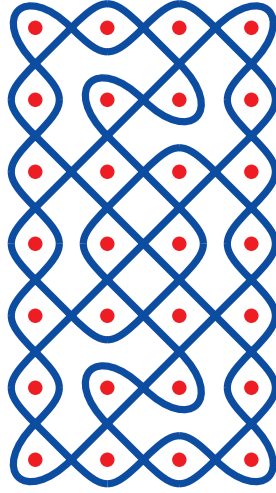
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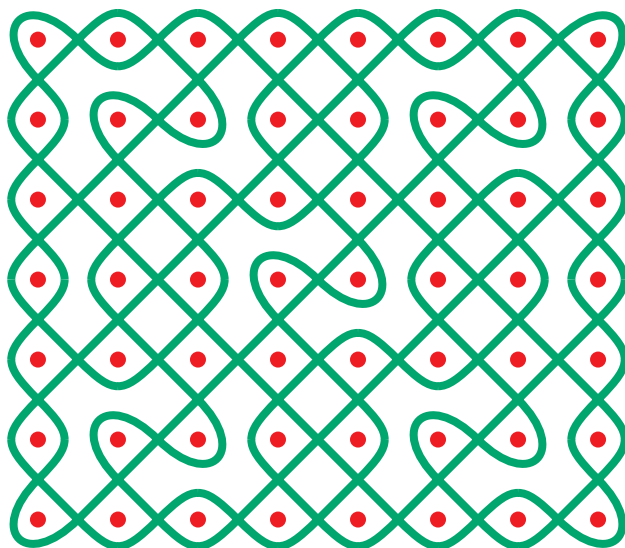
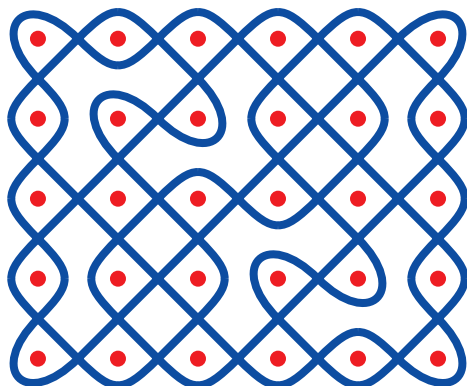
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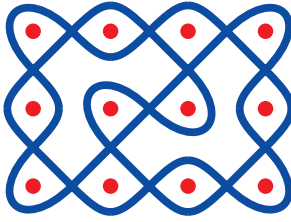


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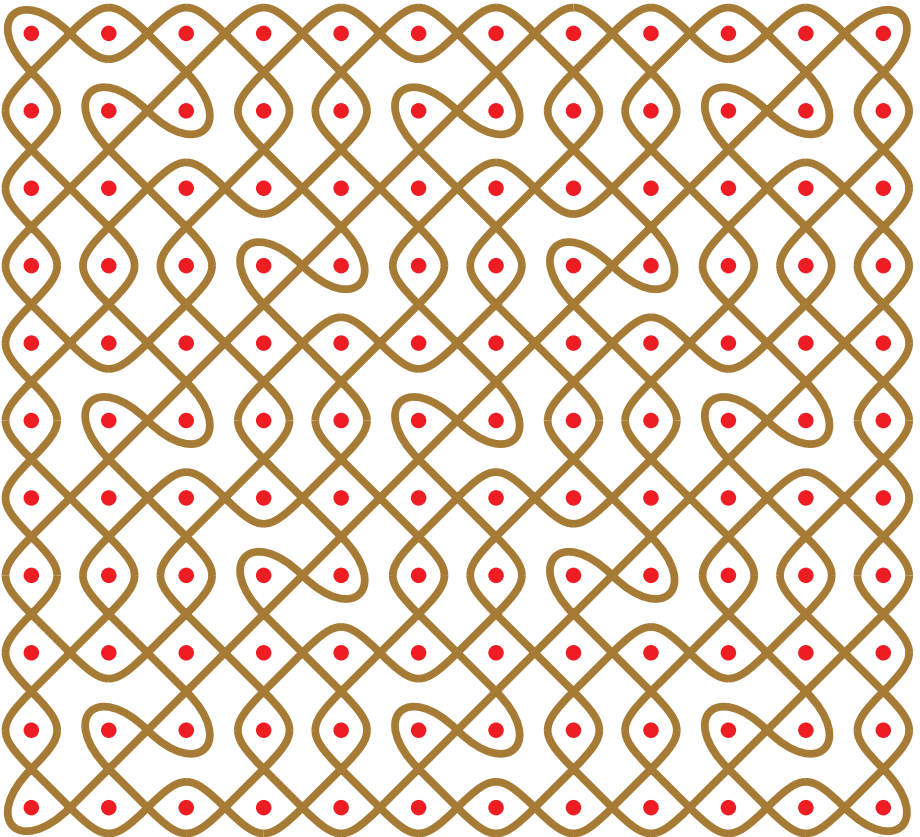
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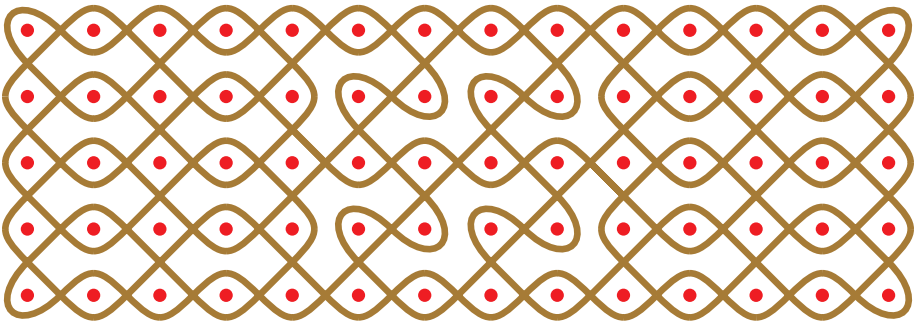
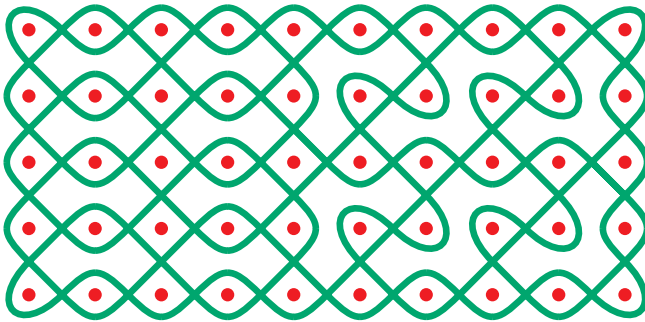
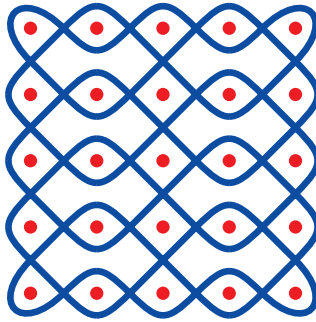
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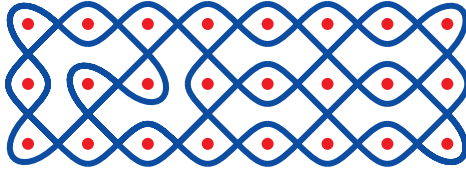


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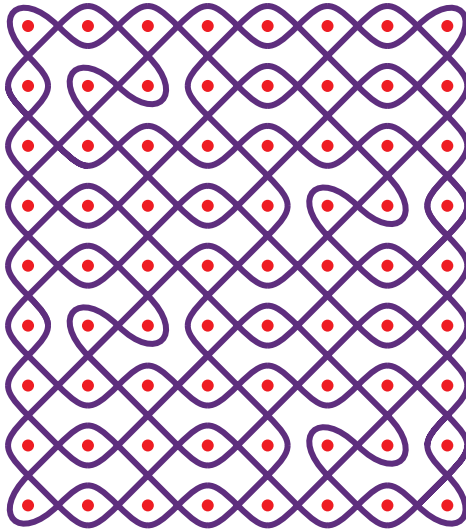
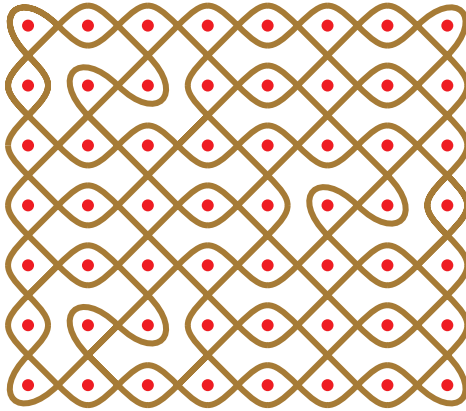


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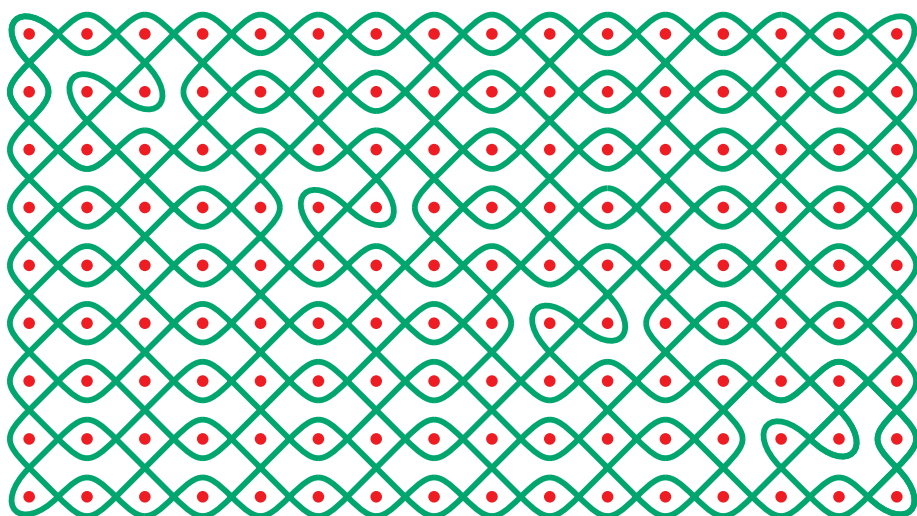
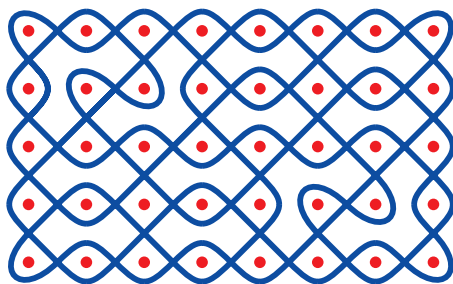
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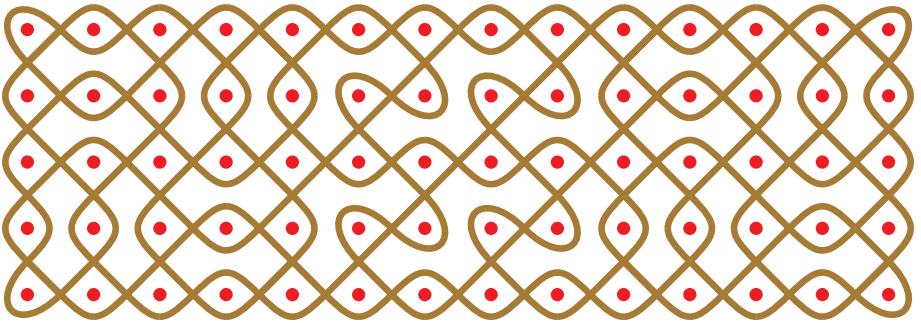
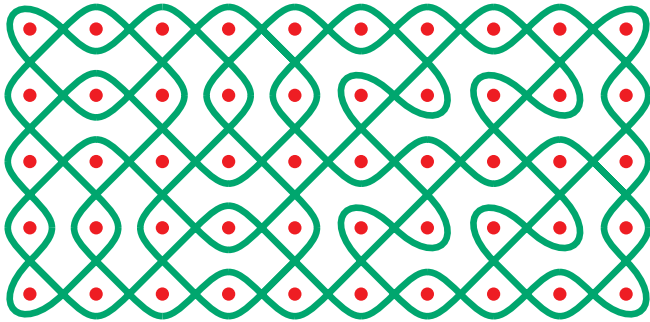
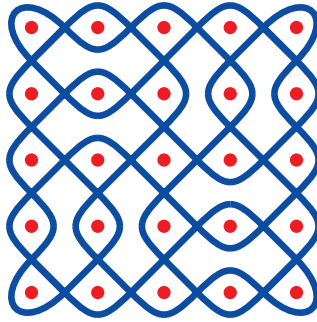
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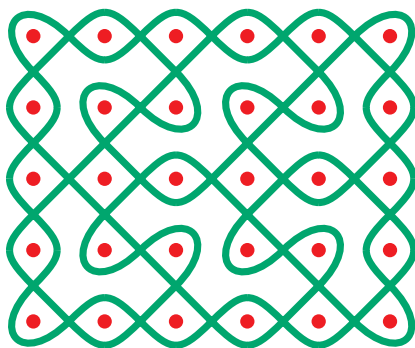
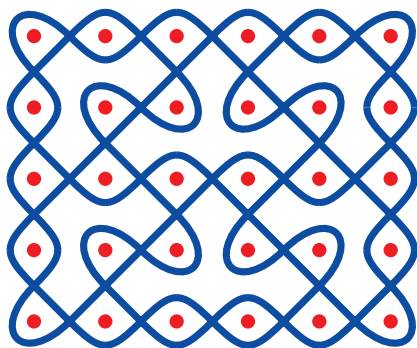


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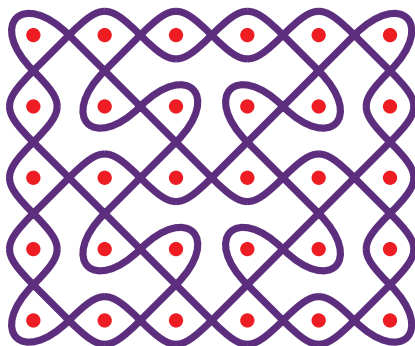


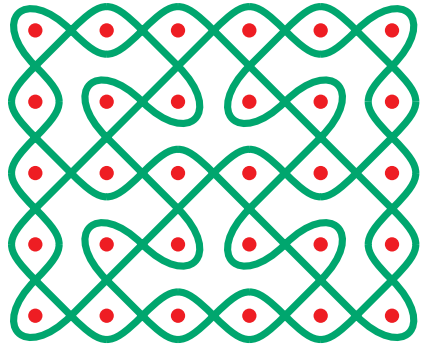
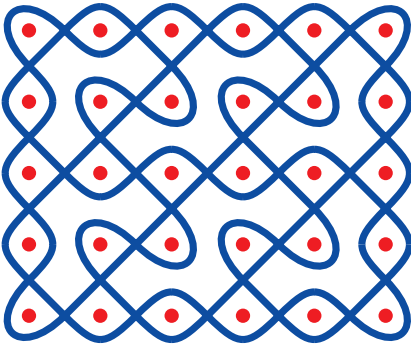
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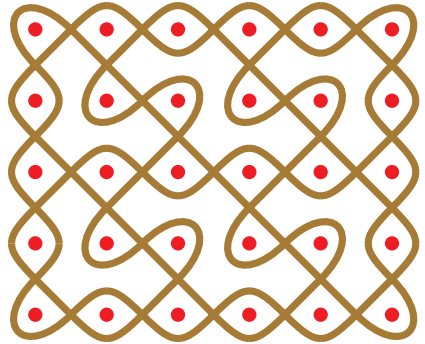


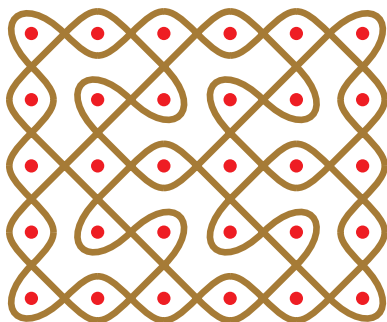
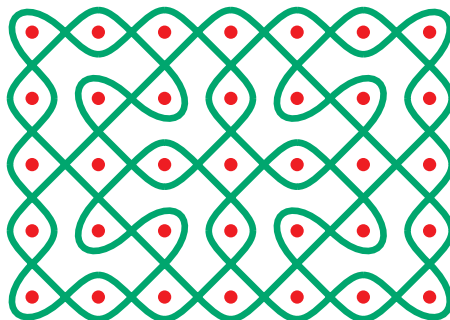
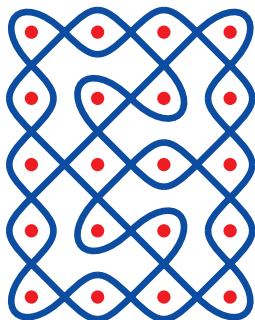
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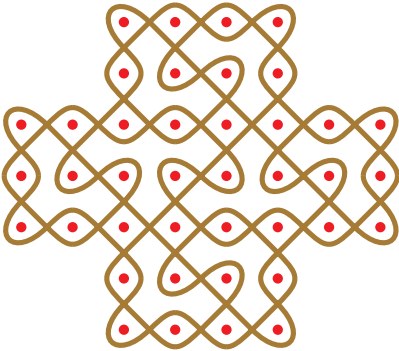
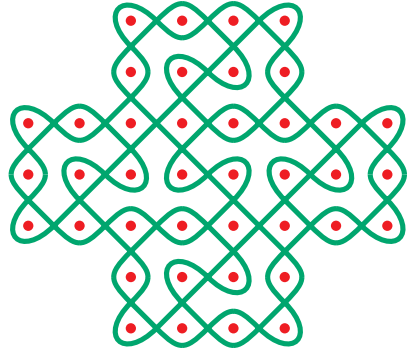
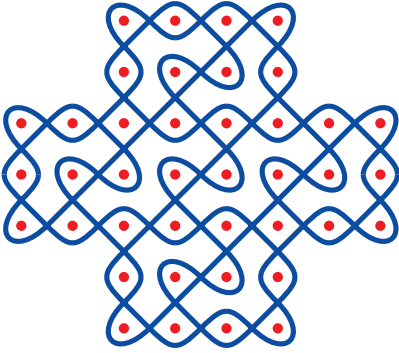


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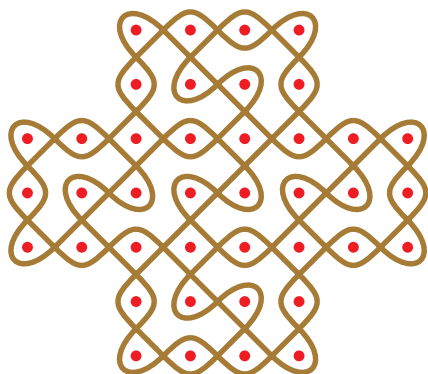
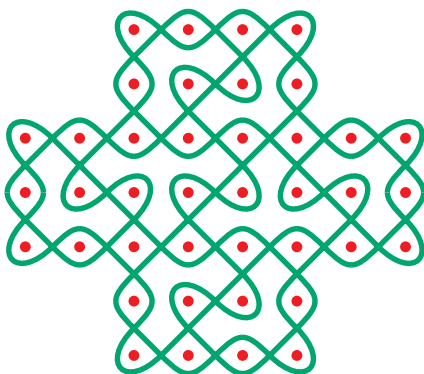
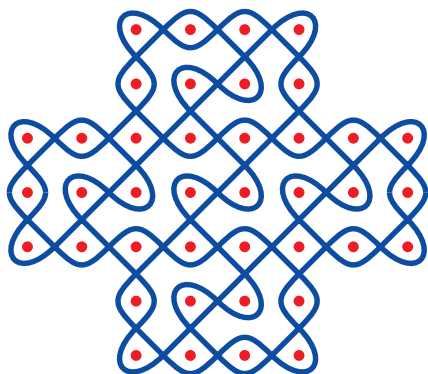




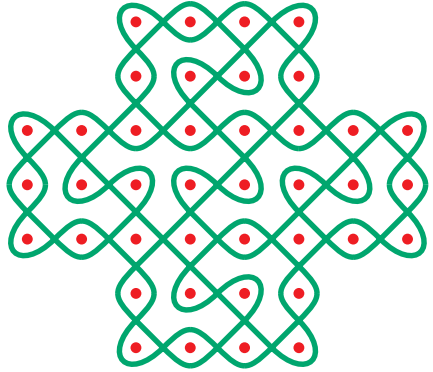
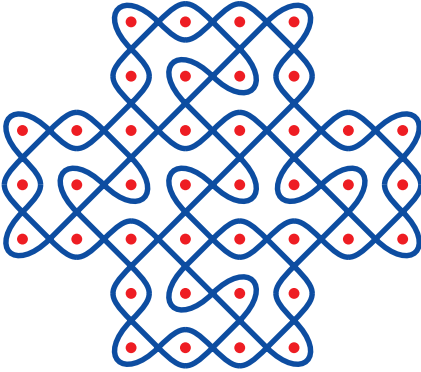
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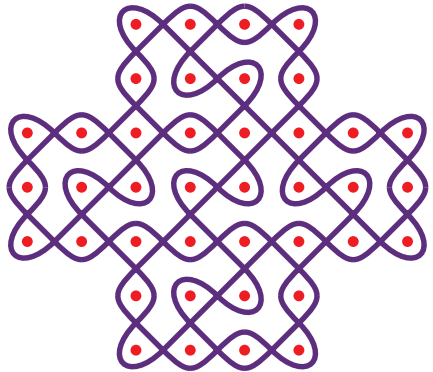
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CAPÍTULO 3

INVENTE A SUA PRÓPRIA SEQUÊNCIA DE *LUSONA*

O desenho Cokwe na Figura 1 representa três pássaros a voar.
O leitor consegue ver cada um dos pássaros?

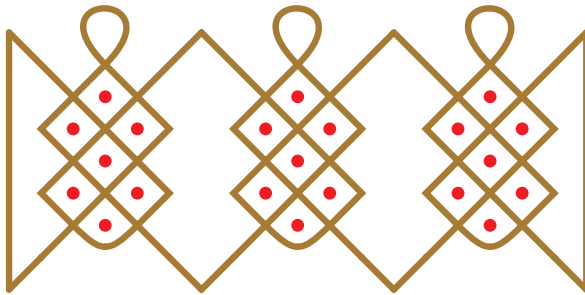


Figura 1

Copie o desenho em papel transparente. Como vê, o desenho é composto por uma única linha fechada.

Pode imaginar uma série de *sona* em que o *lusona* dado aparece como um dos seus elementos?

Sugestão:

Tente desenhar um só pássaro.

Tente desenhar dois pássaros em voo, etc.

O *lusona* apresentado na Figura 1 constitui o terceiro elemento da série apresentada na Figura 2.

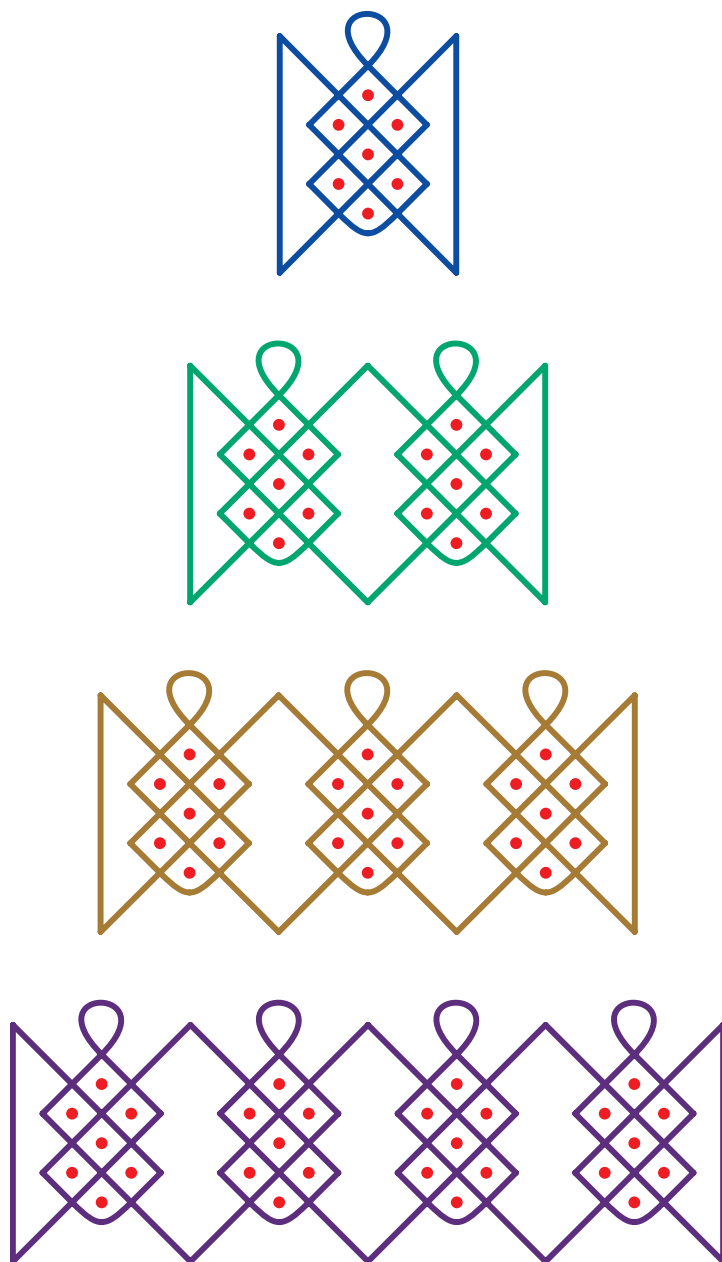


Figura 2

Esta sequência não é a única em que o *lusona* dado aparece com um dos seus elementos. O mesmo *lusona* aparece na sequência apresentada na Figura 3. Desta vez como segundo elemento.

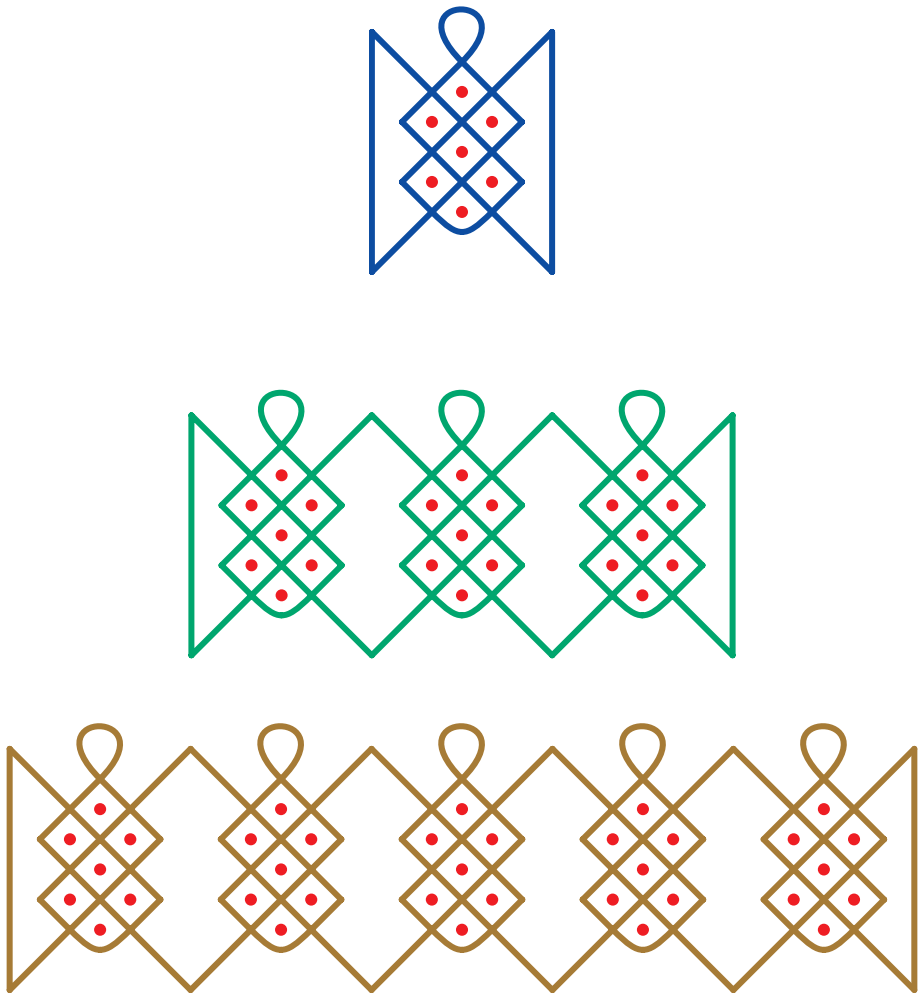


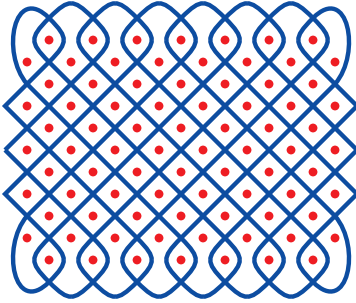
Figura 3

Nas páginas que se seguem apresento alguns *sona* tradicionais (páginas 129-131) e alguns padrões novos (páginas 132-137). Cada um deles é composto por uma única linha.

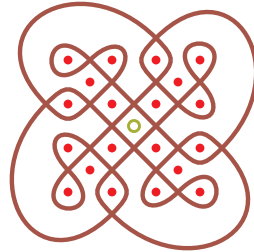
Para cada um dos padrões dados, tente encontrar uma série em que ele aparece como um dos seus elementos. Mais uma vez, todos os desenhos têm de ser compostos por uma única linha fechada.

Exemplos de SONA tradicionais

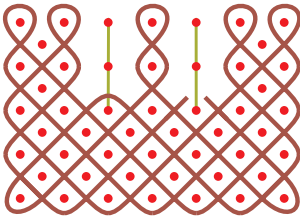
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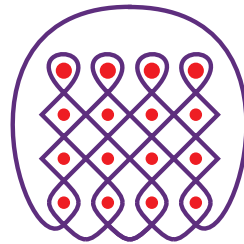
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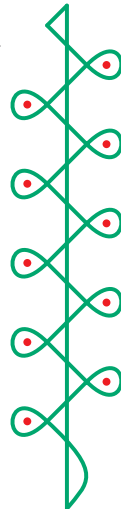
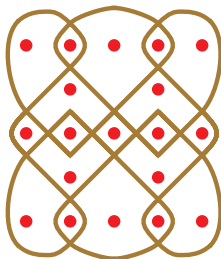
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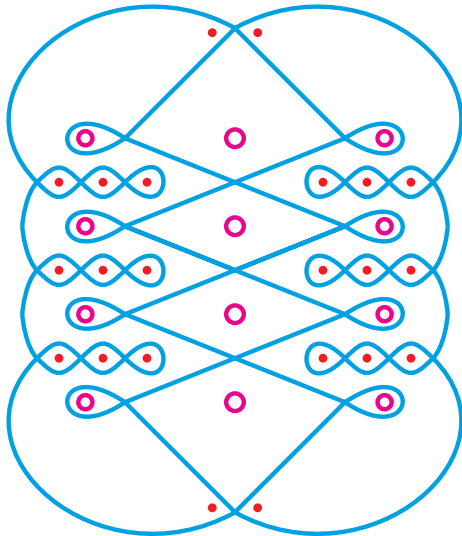
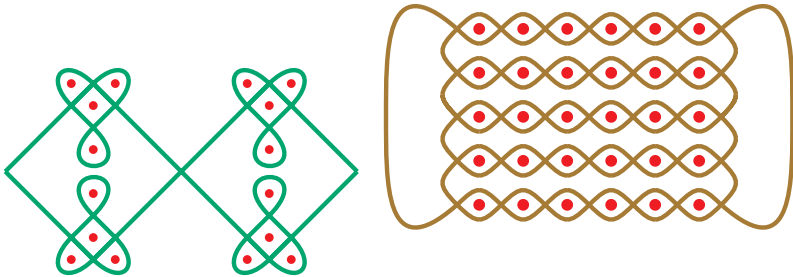
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Exemplos de SONA tradicionais

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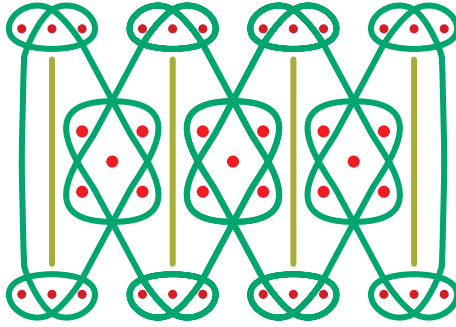


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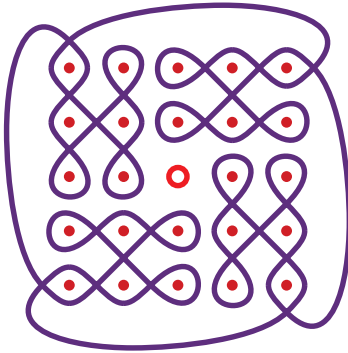


Exemplos de SONA tradicionais

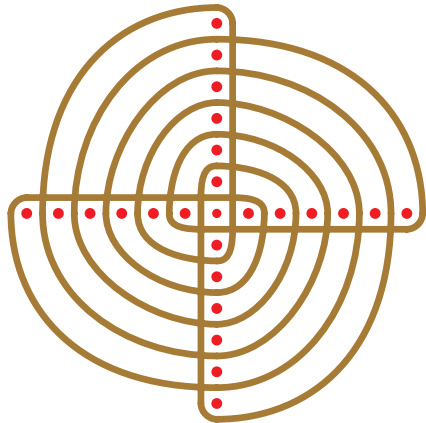
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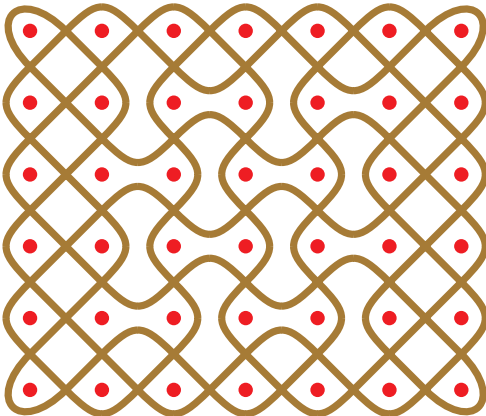
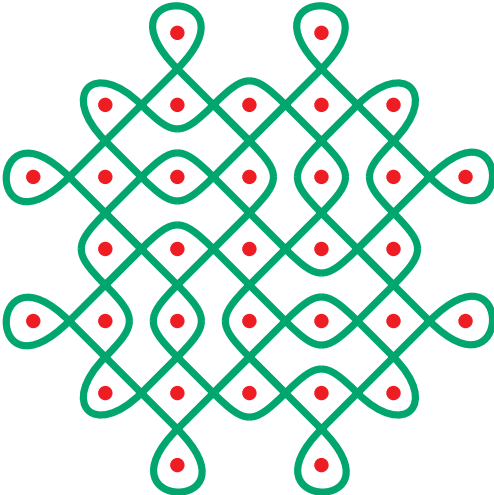
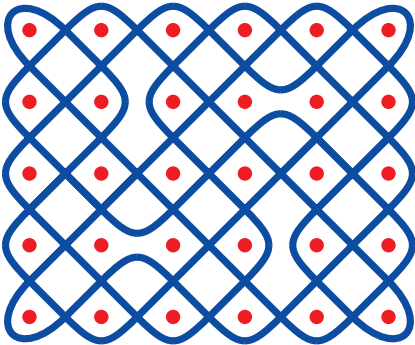
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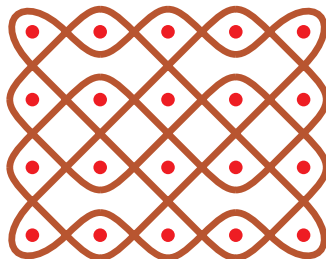
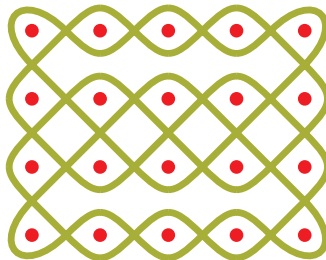
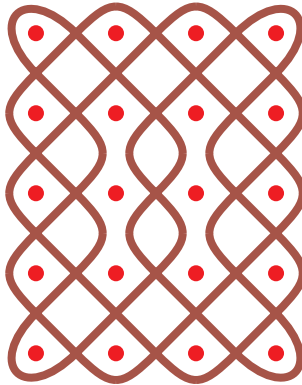
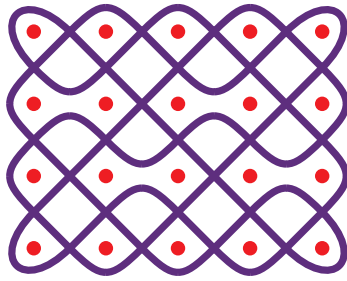


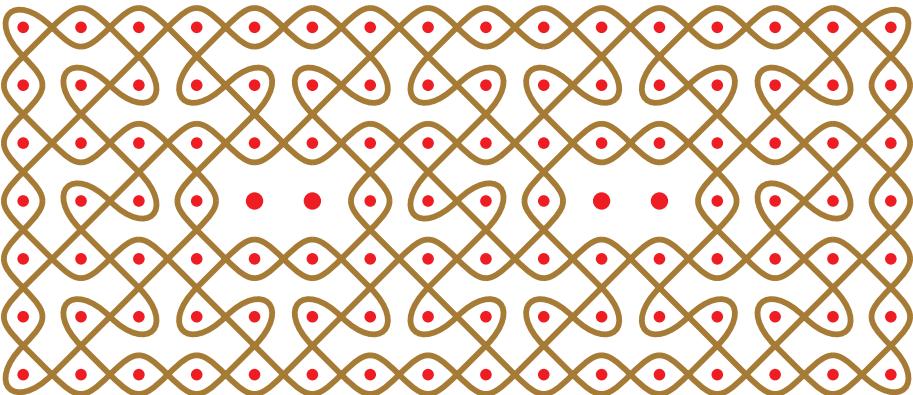
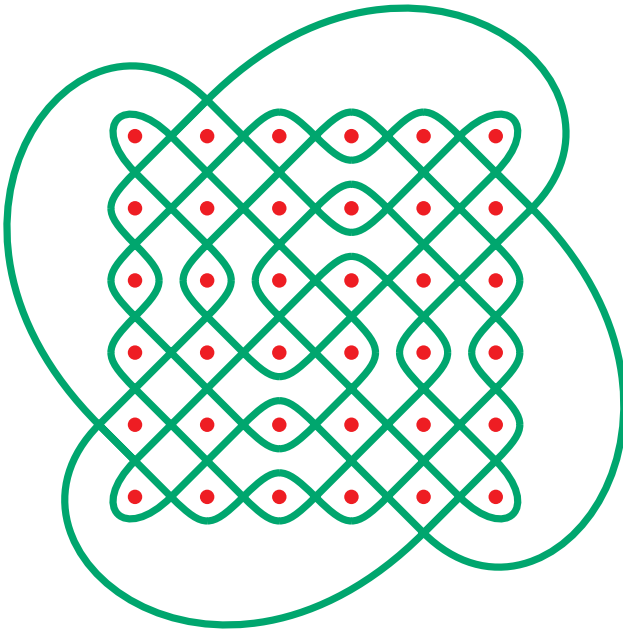
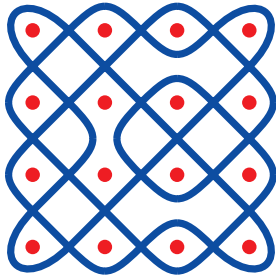
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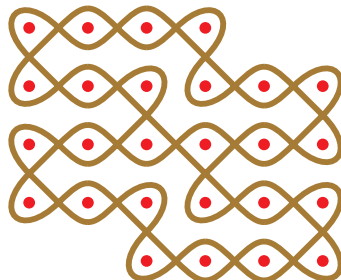
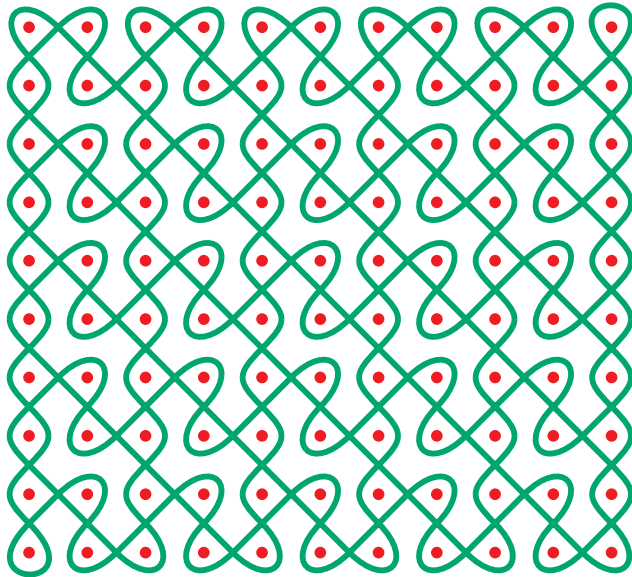
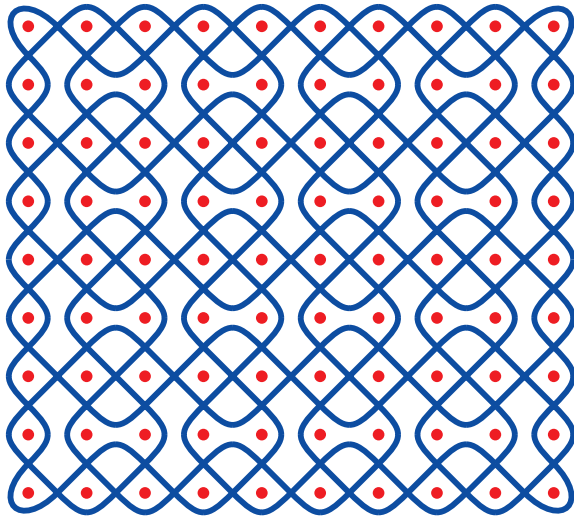


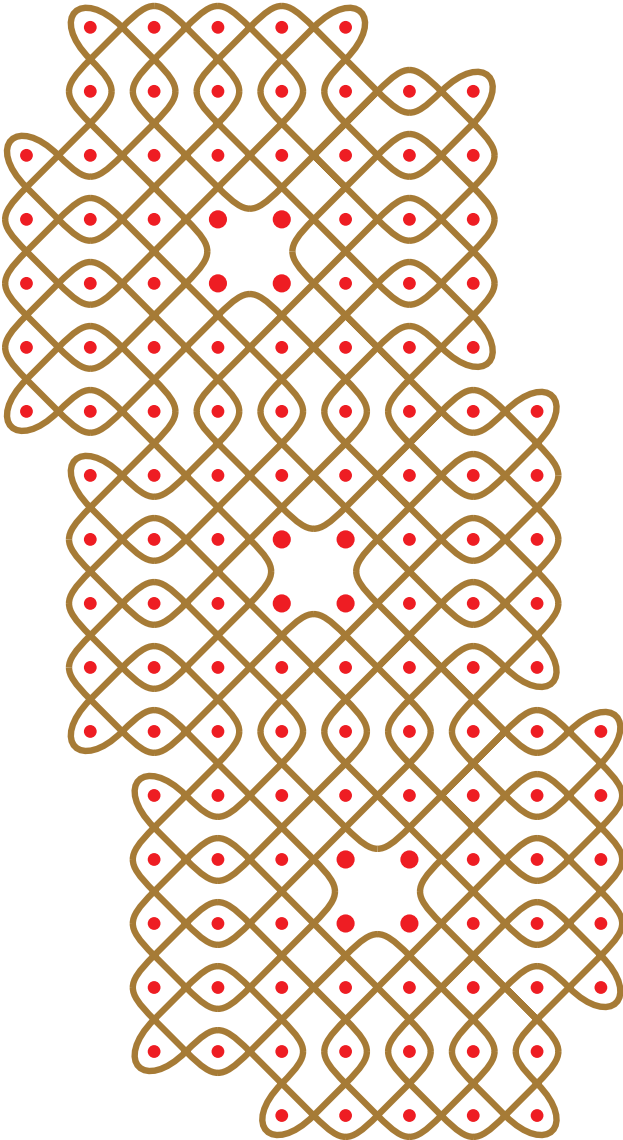
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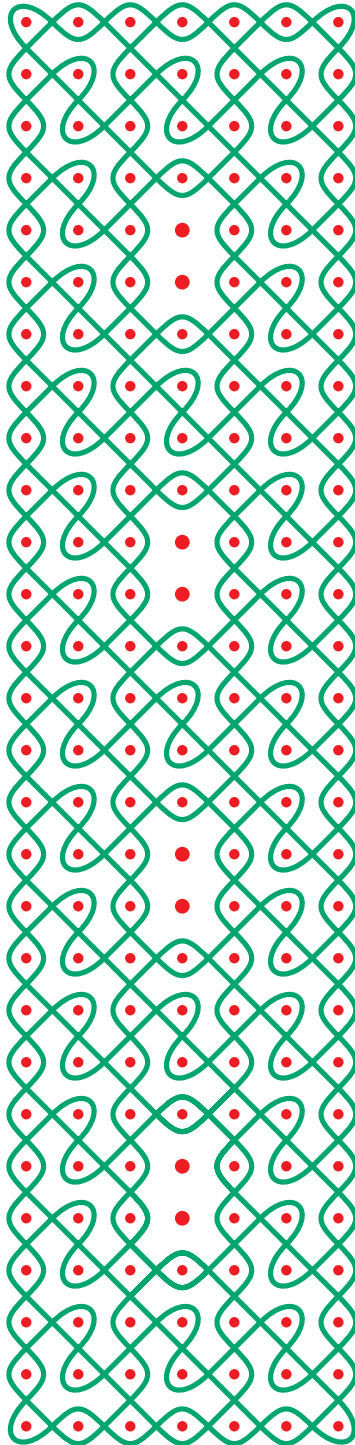












CAPÍTULO 4

INVENTE SONA E SEQUÊNCIAS DE SONA

Convida-se o leitor a inventar os seus próprios *(lu)sona* e sequências de *sona*.

Se quiser, envie as suas propostas ao autor. Propostas interessantes poderão ser incluídas num livro novo. Apresente também os seus comentários sobre ***Lusona: Recreações Geométricas de África*** ***Africa. Problemas e Soluções***.

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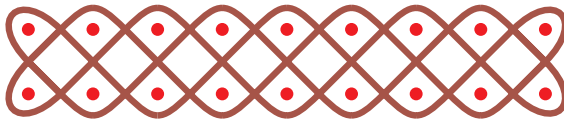
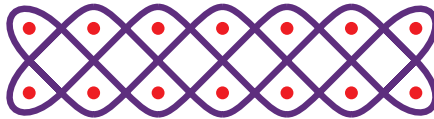
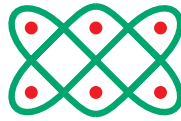
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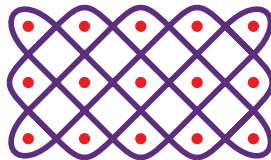
Jaritz, Wolfgang: Über Bahnen auf Billardtischen – oder: Eine mathematische Untersuchung von Ideogrammen Angolanischer Herkunft, *Berichte der mathematisch-statistischen Sektion im Forschungs-zentrum Graz*, Graz, 1983, N° 207, 1-22.

SOLUÇÕES

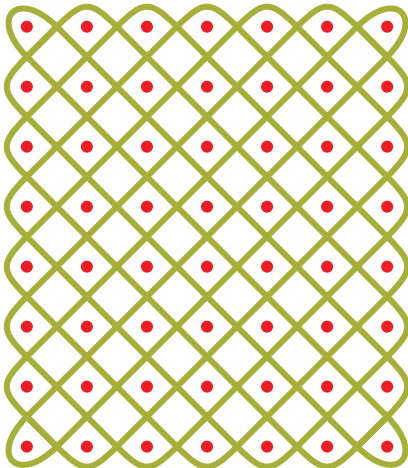
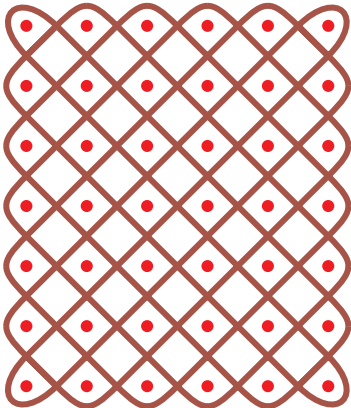
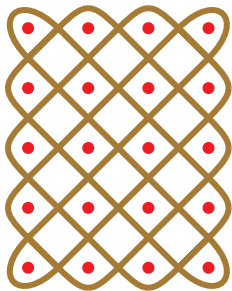
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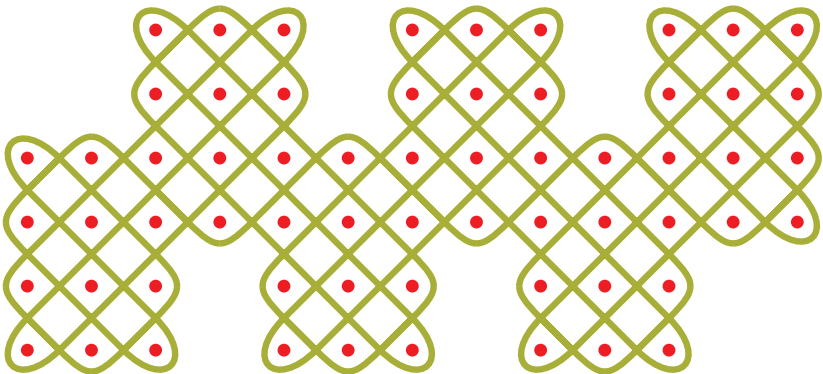
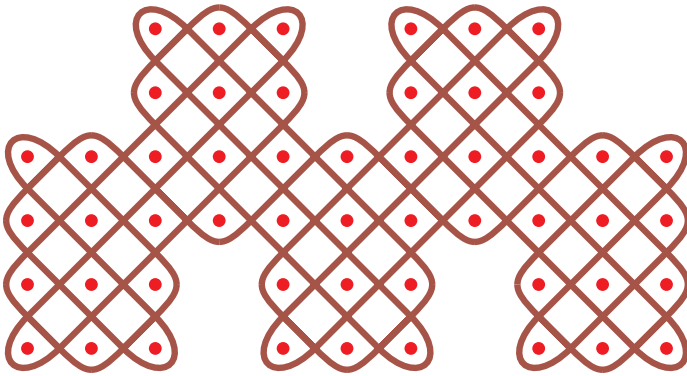
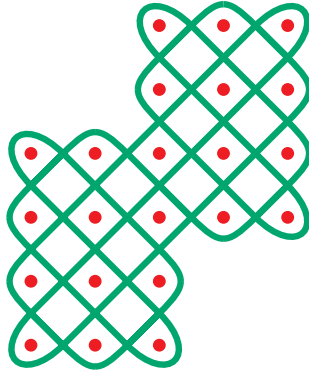


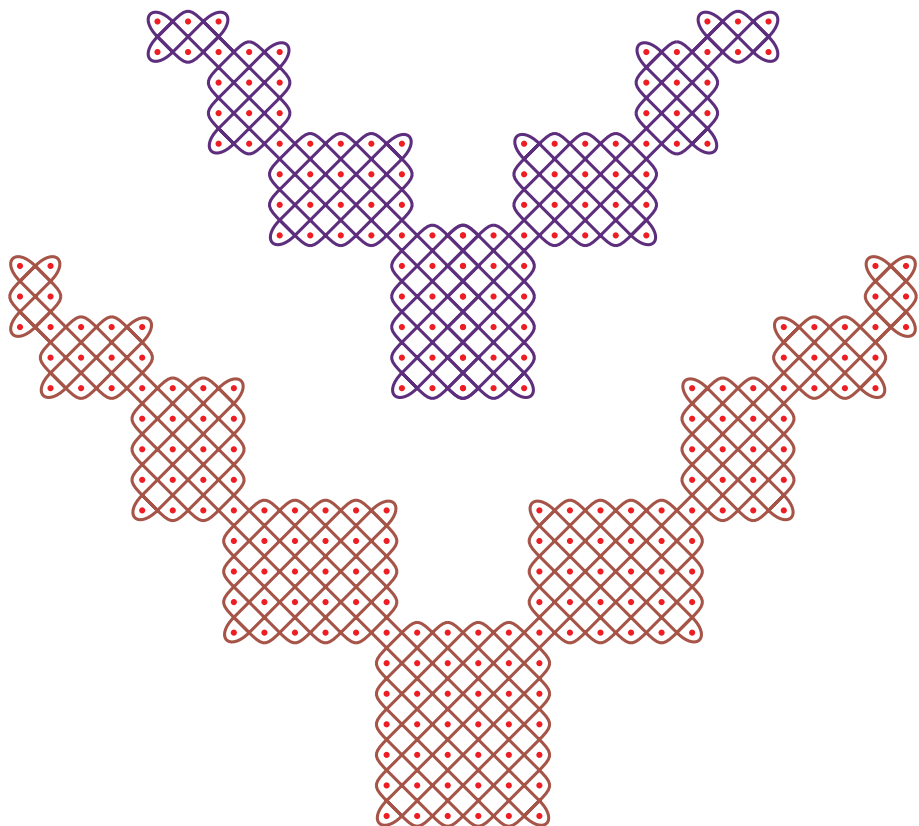
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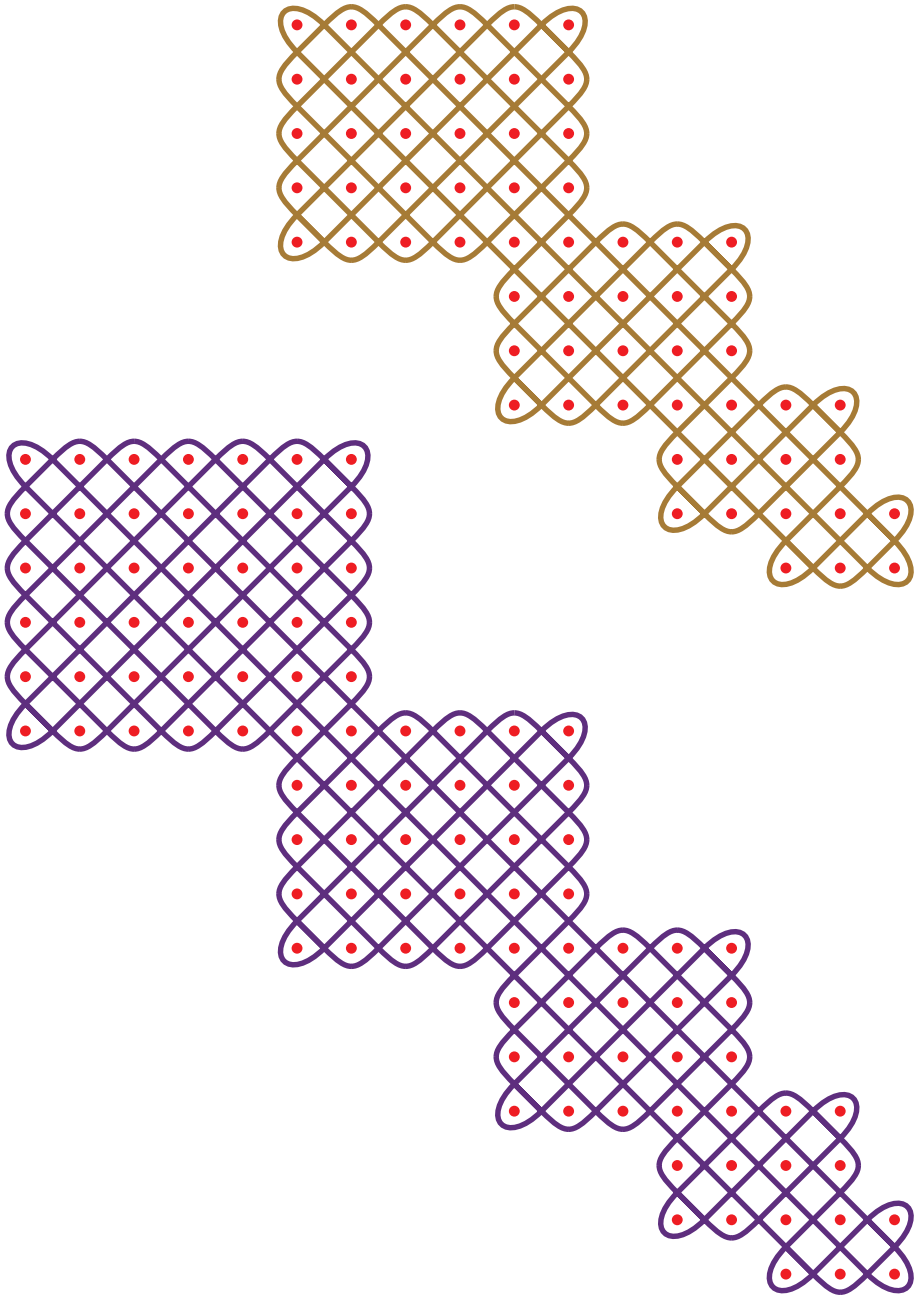


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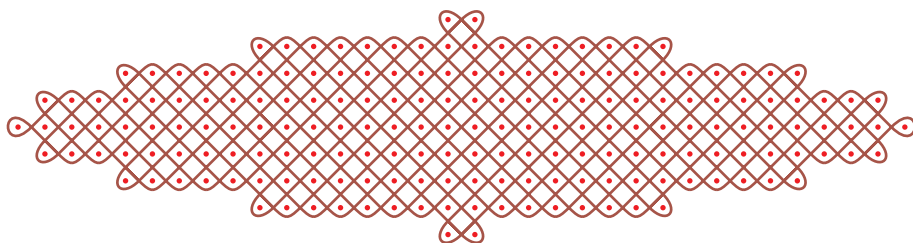




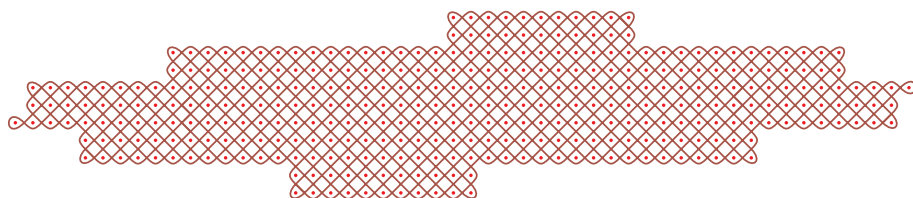
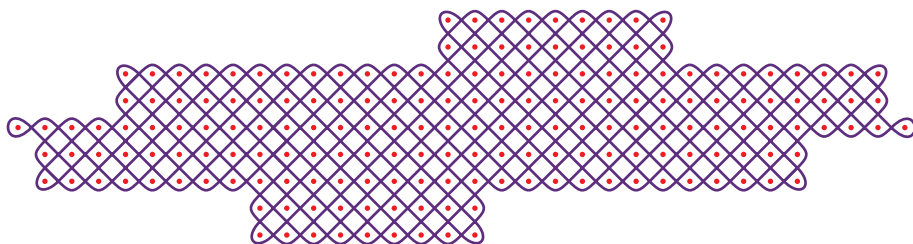


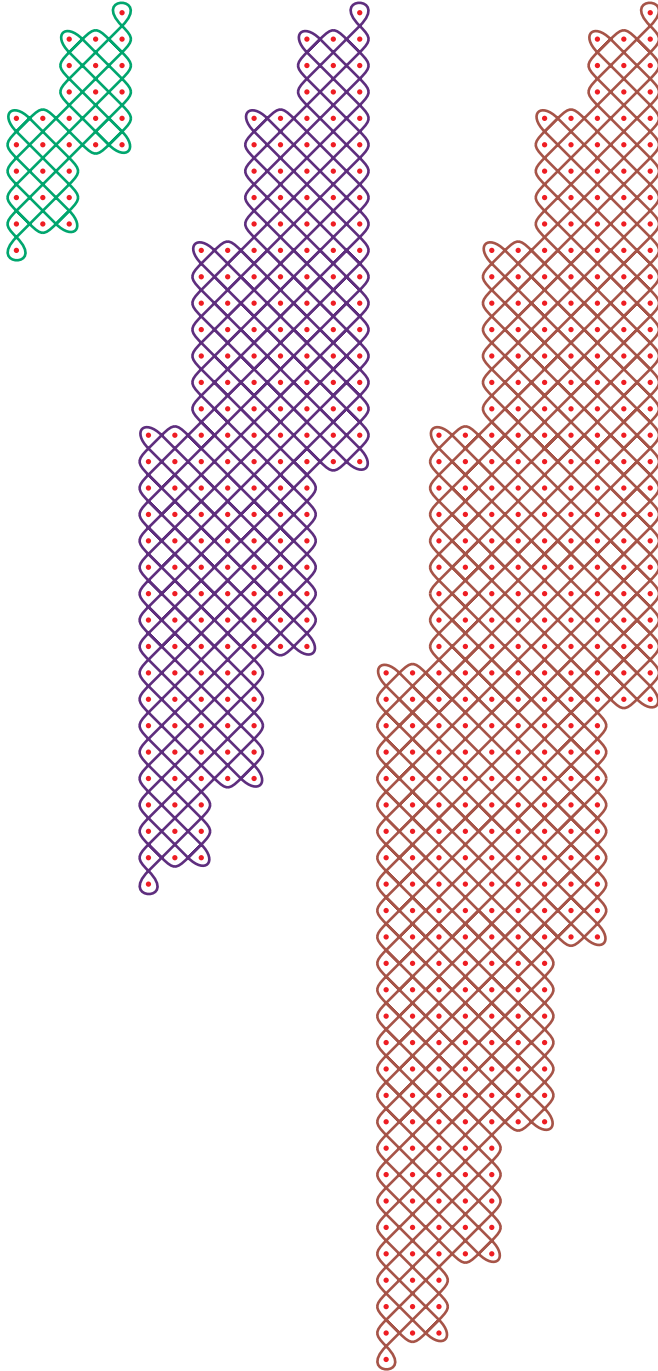


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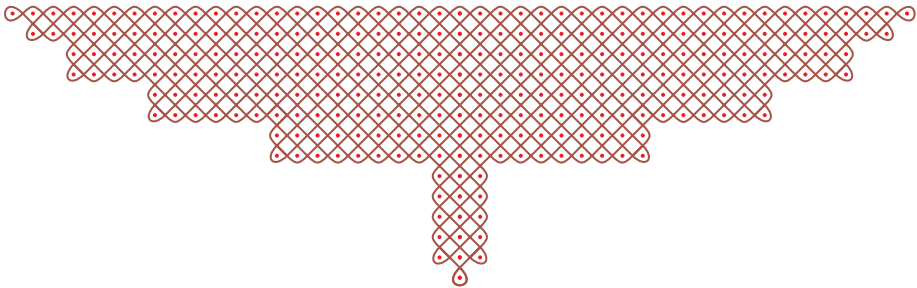
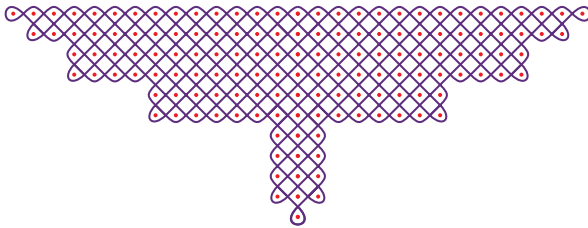


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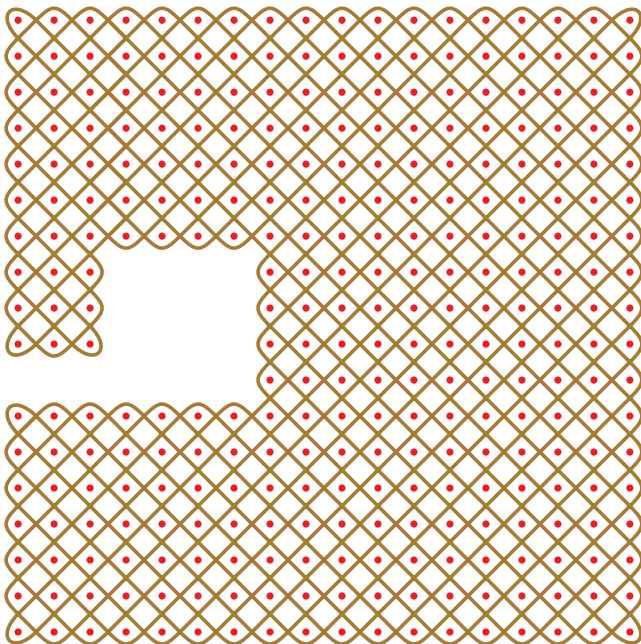


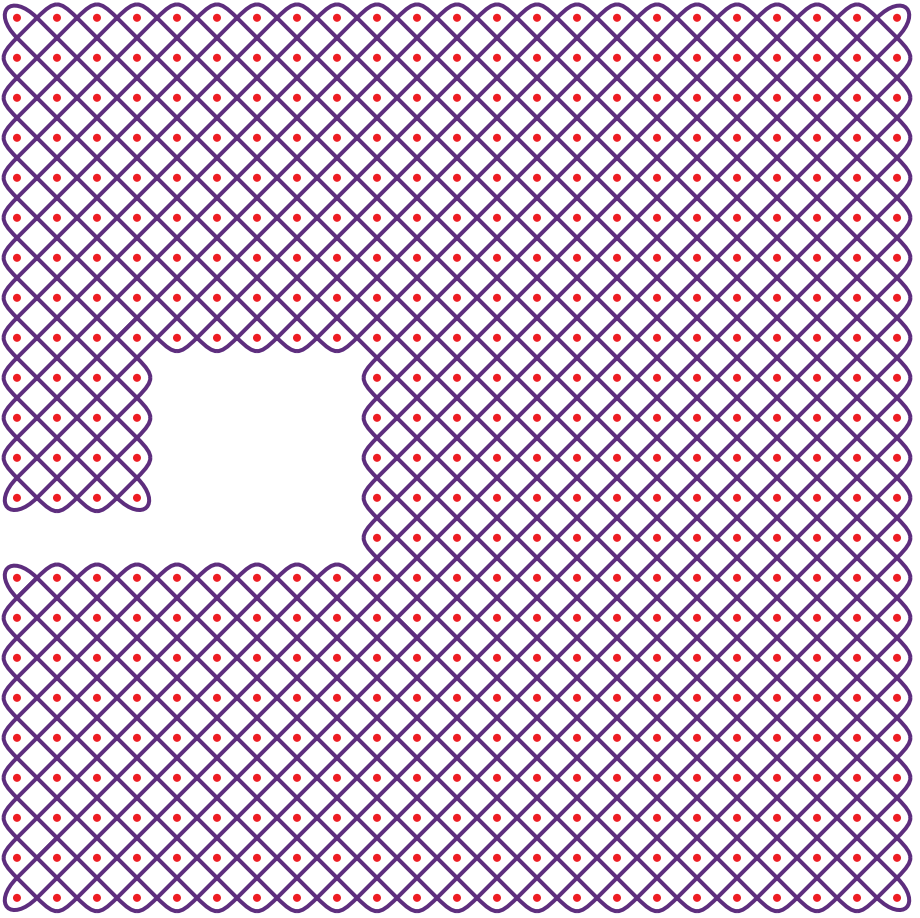


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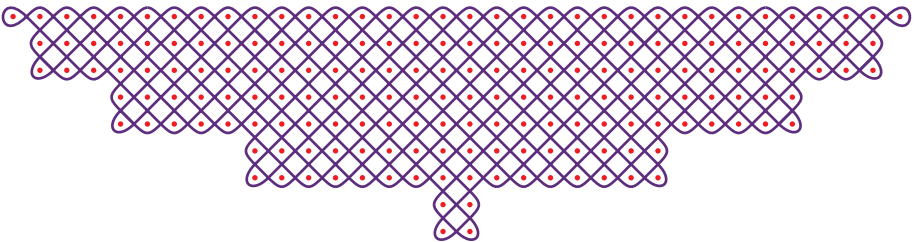
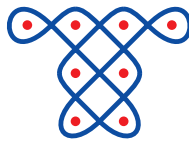


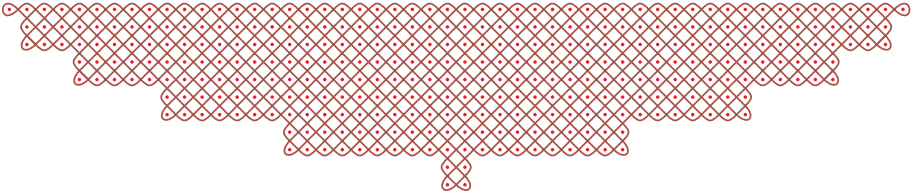
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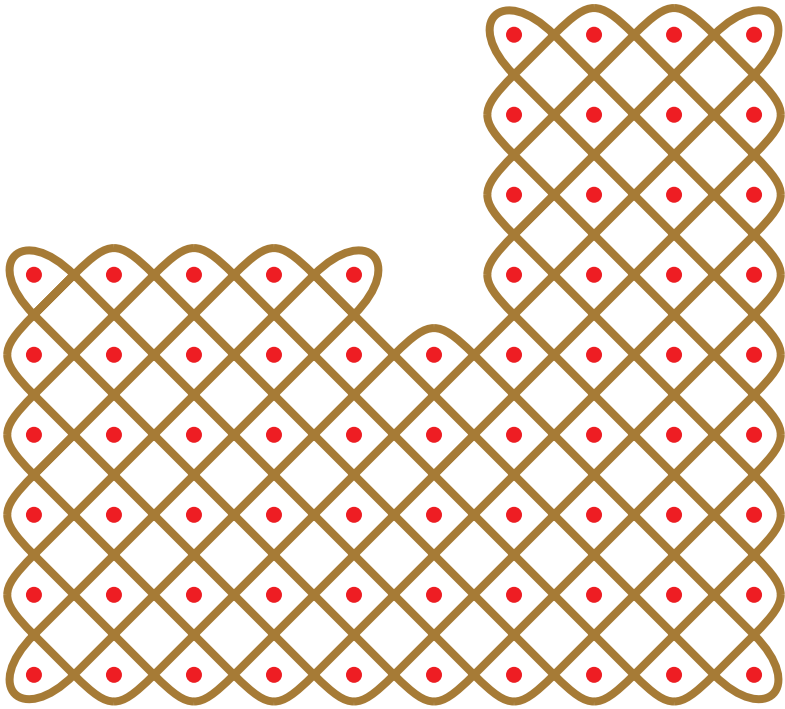


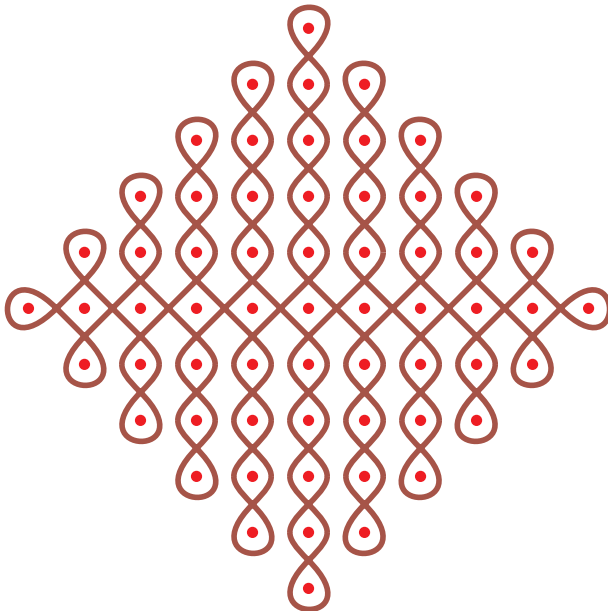
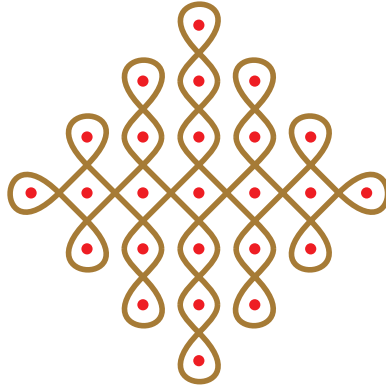
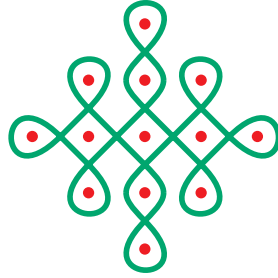
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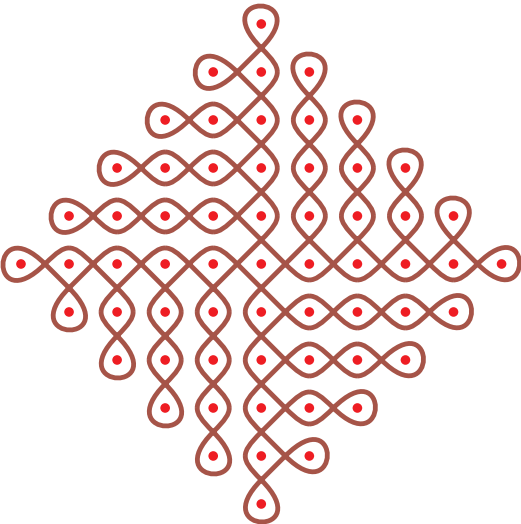
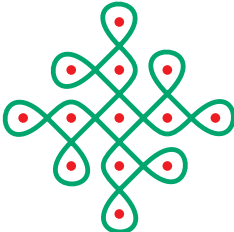
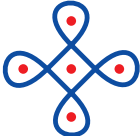


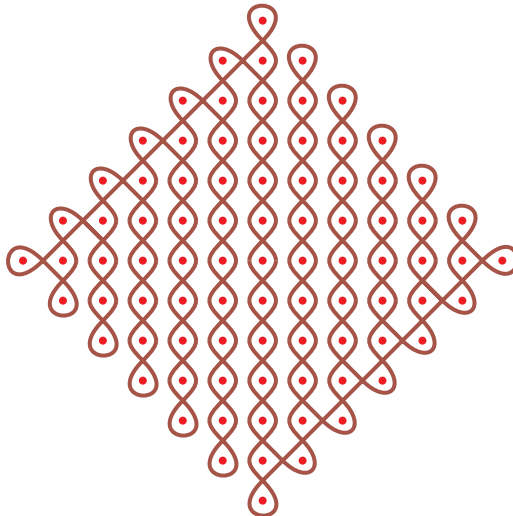
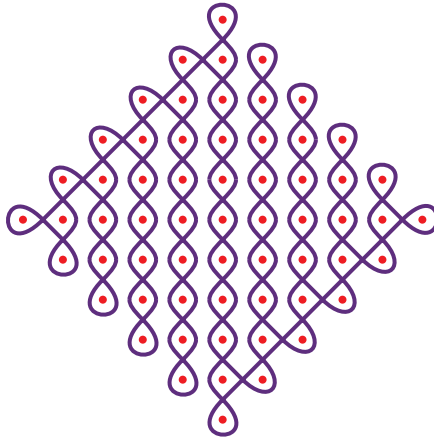
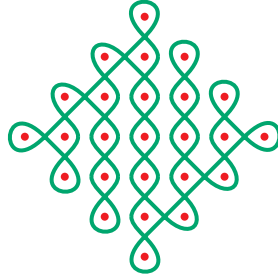


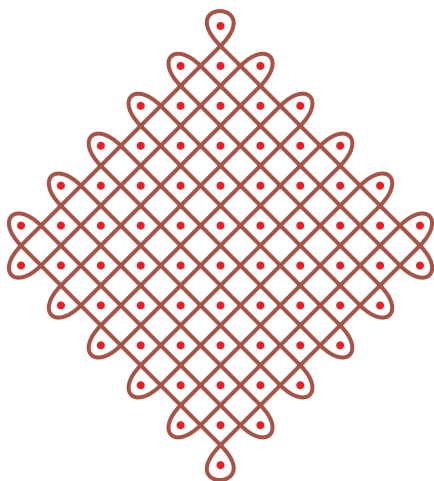
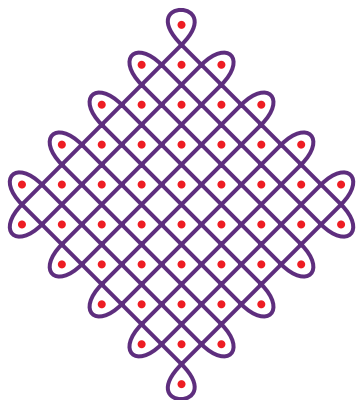
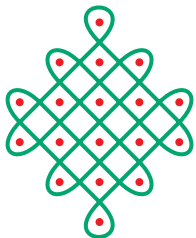
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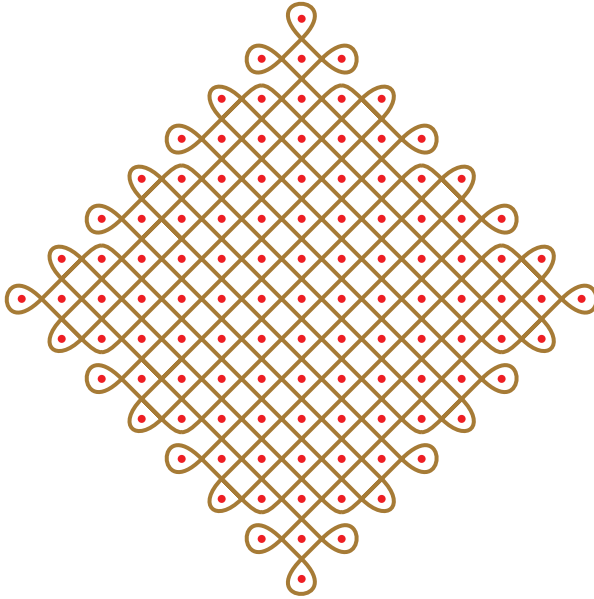




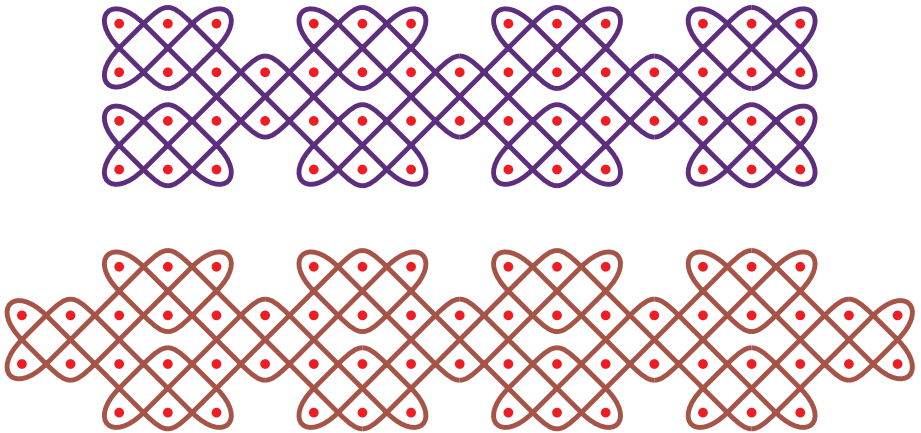




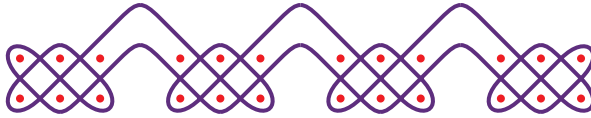
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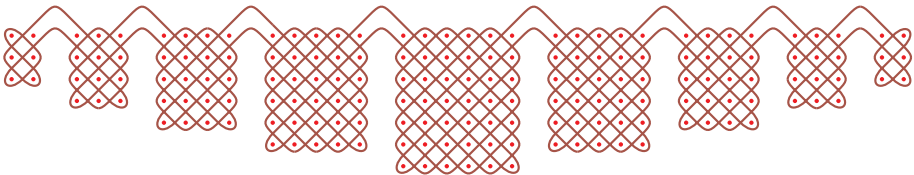
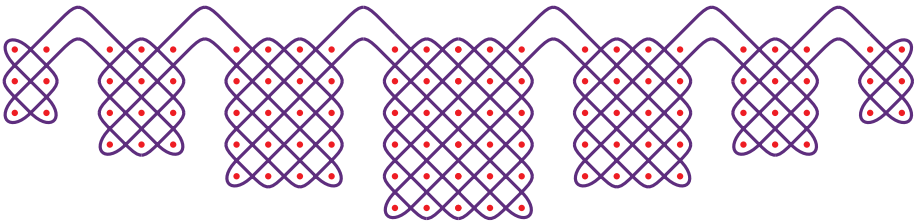
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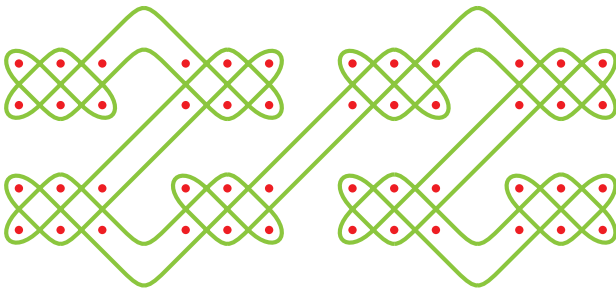
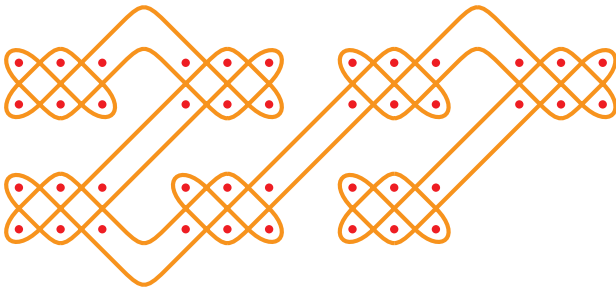
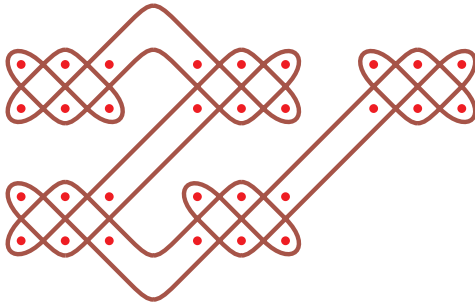
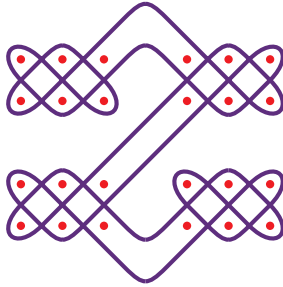


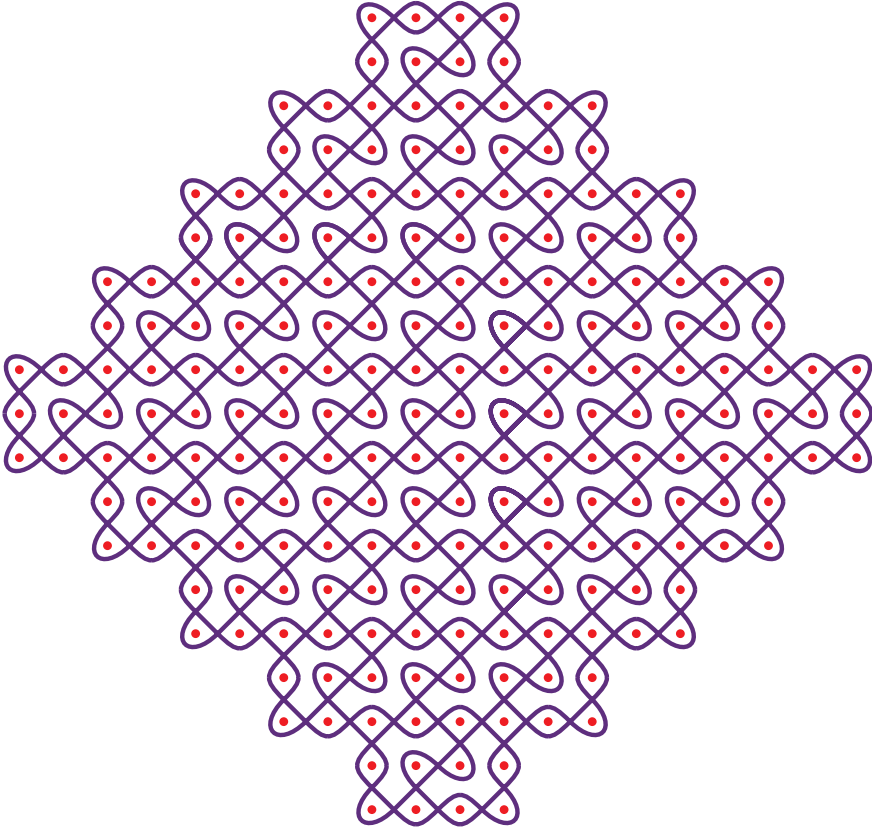
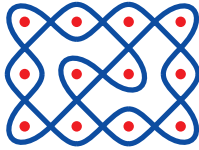
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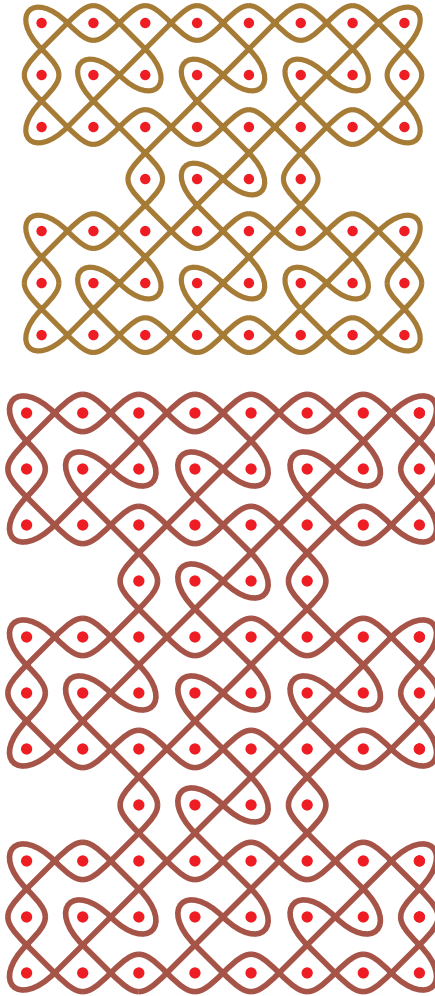
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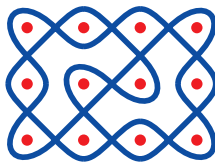


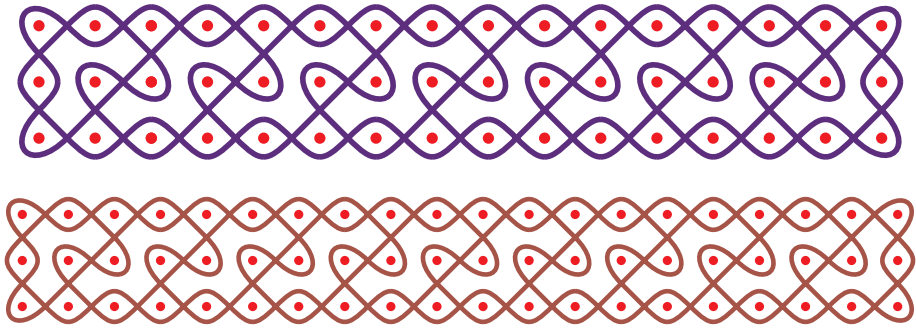


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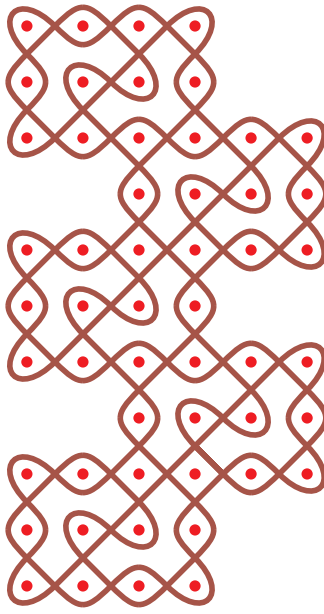
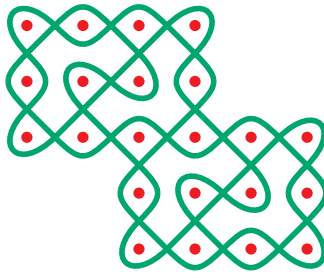


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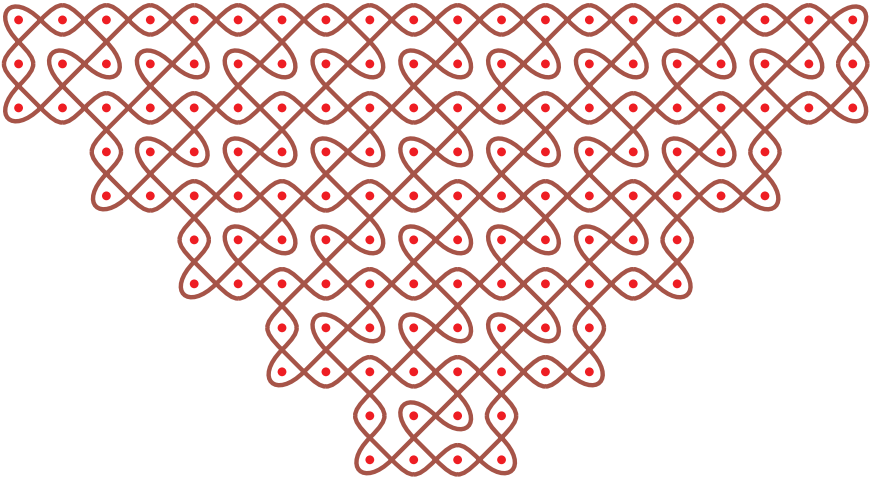




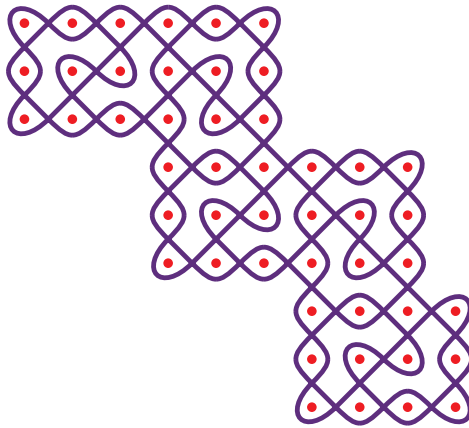
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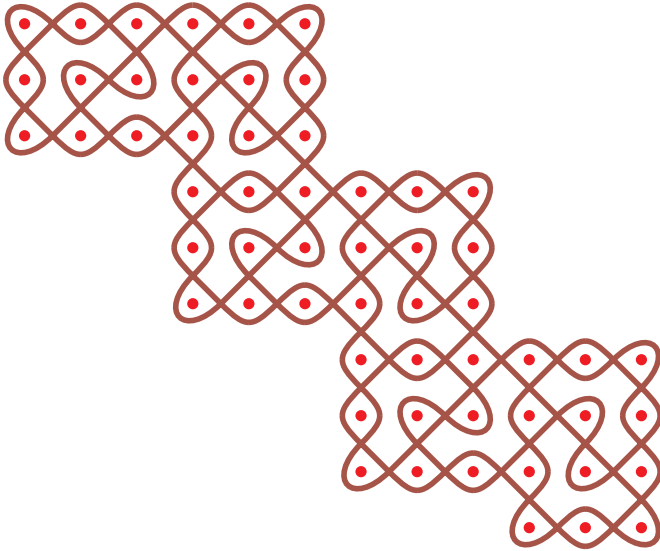


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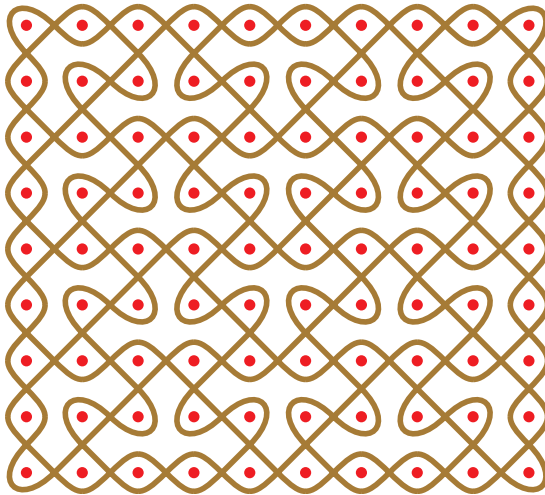


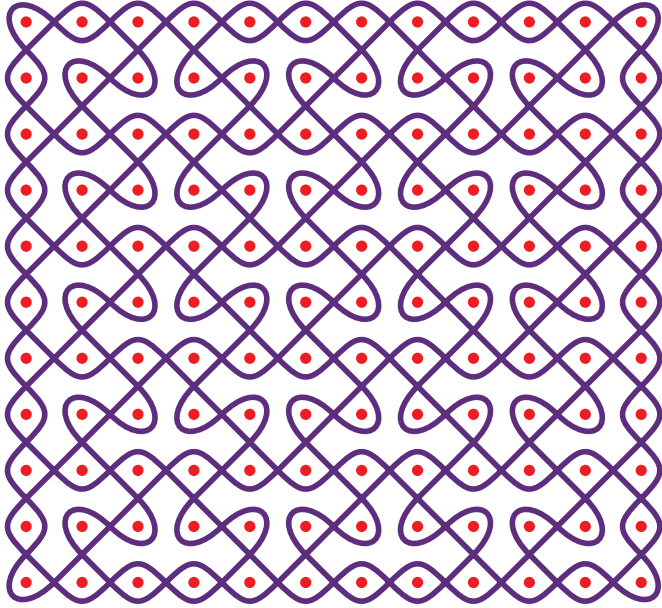
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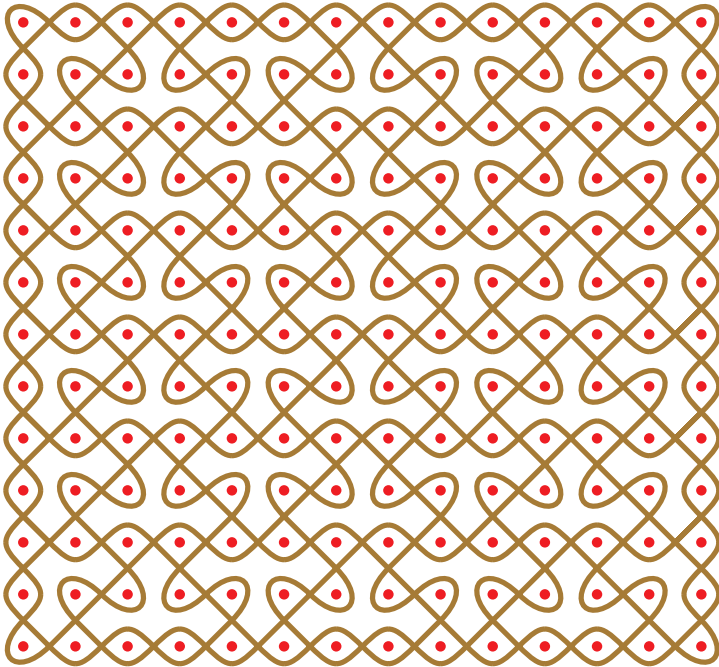


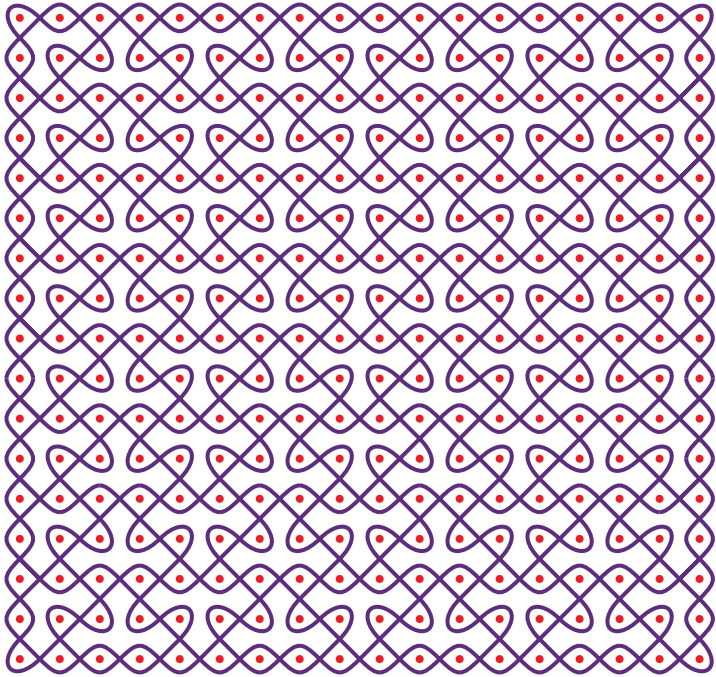
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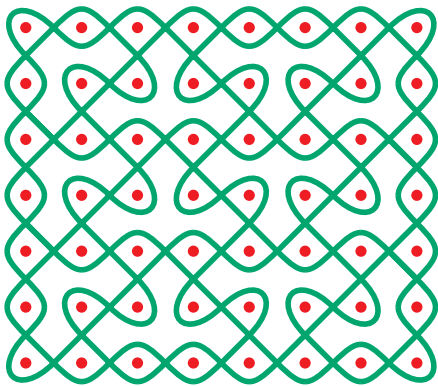


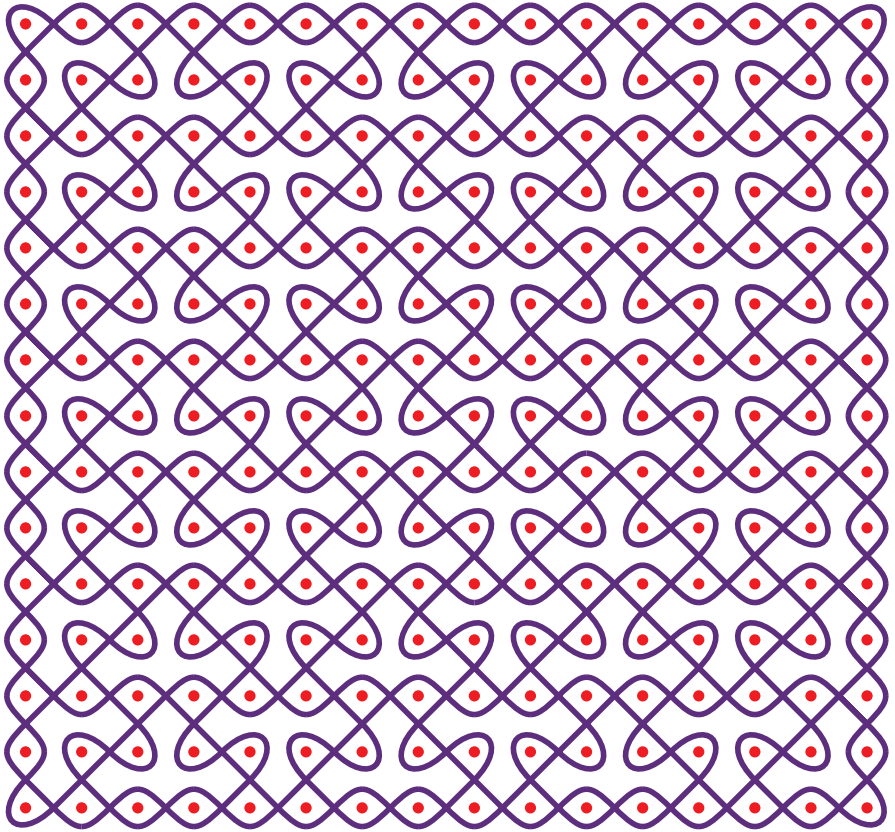
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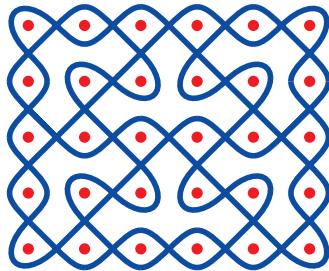


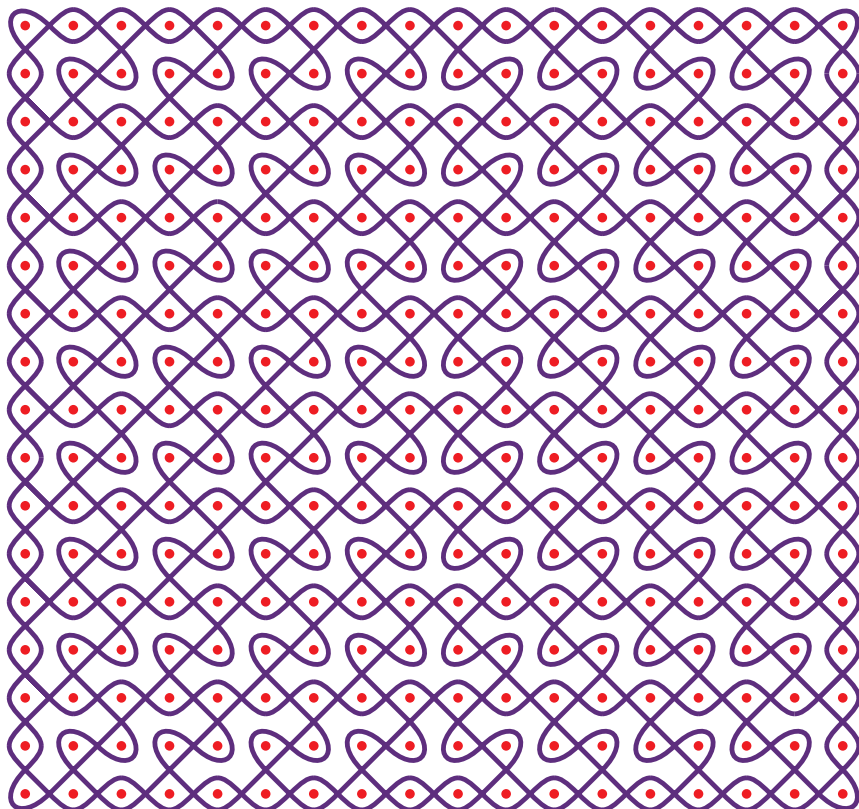
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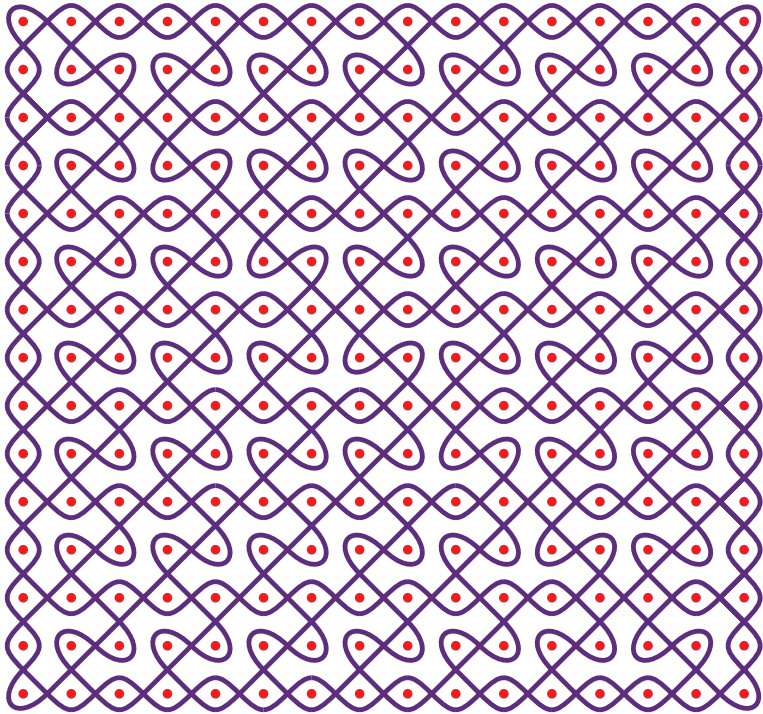


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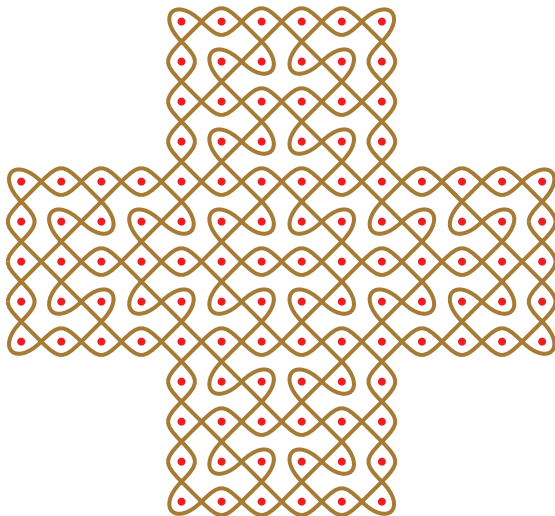


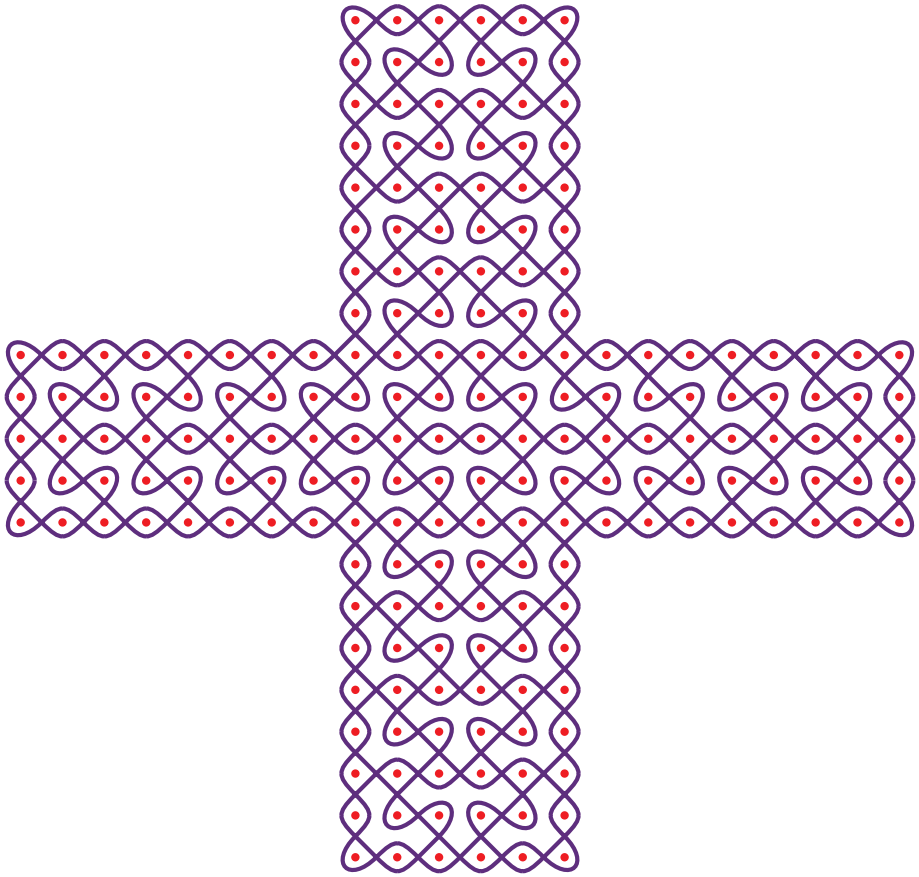


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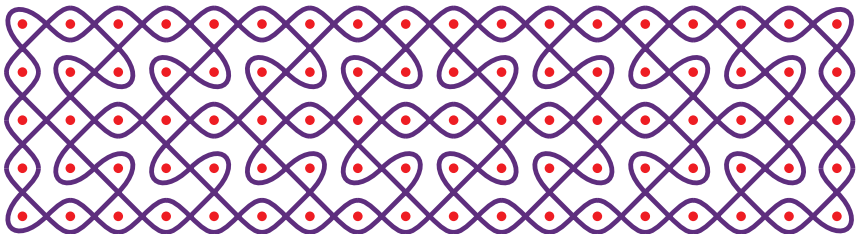
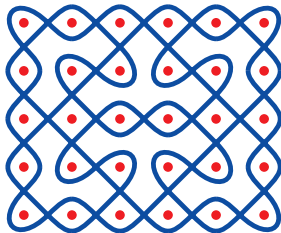


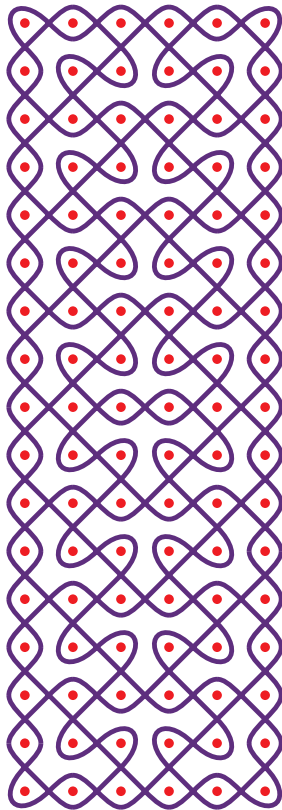
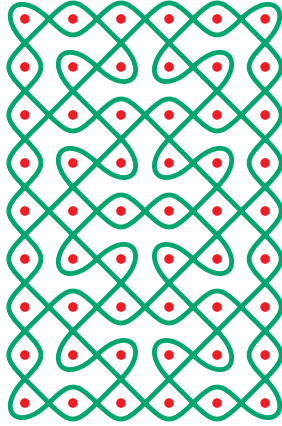
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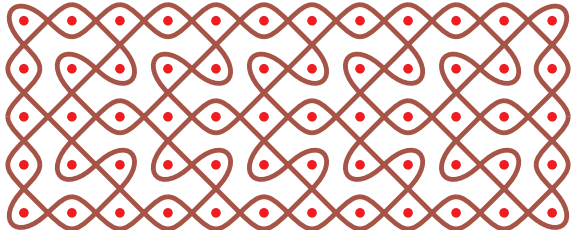
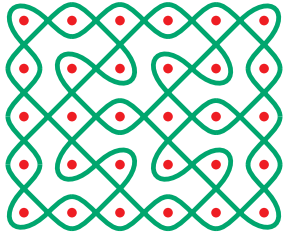


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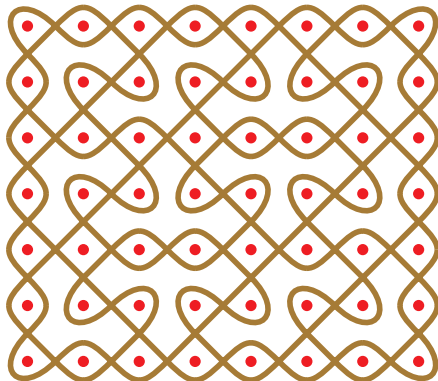
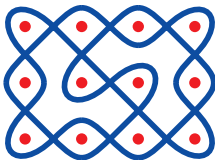


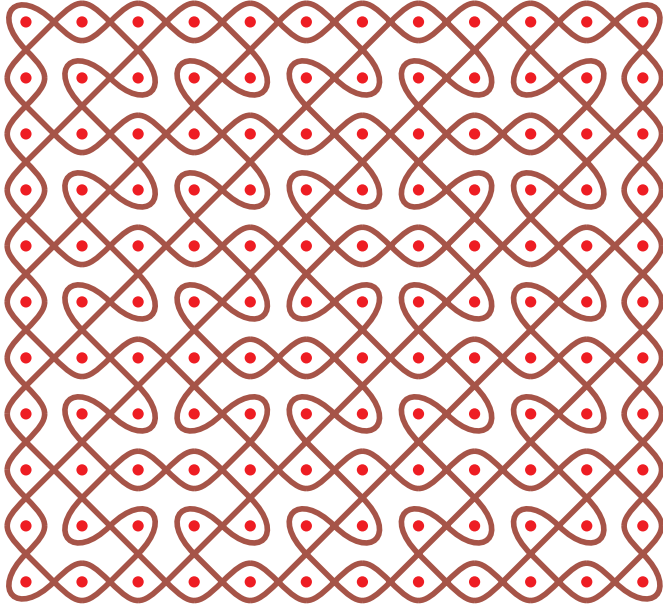


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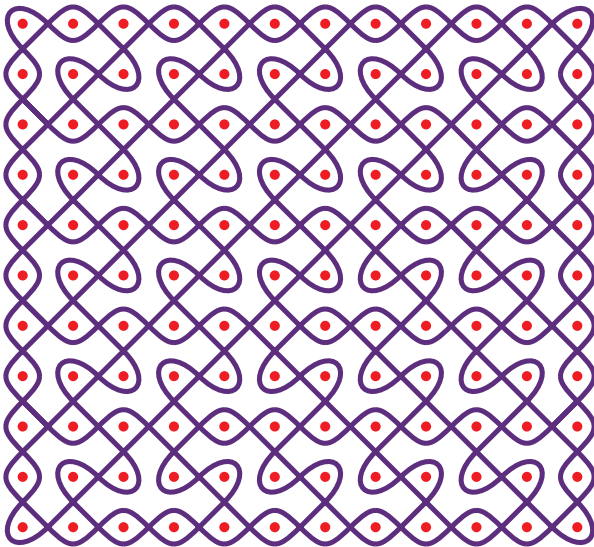
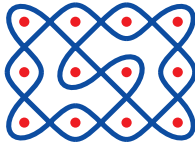


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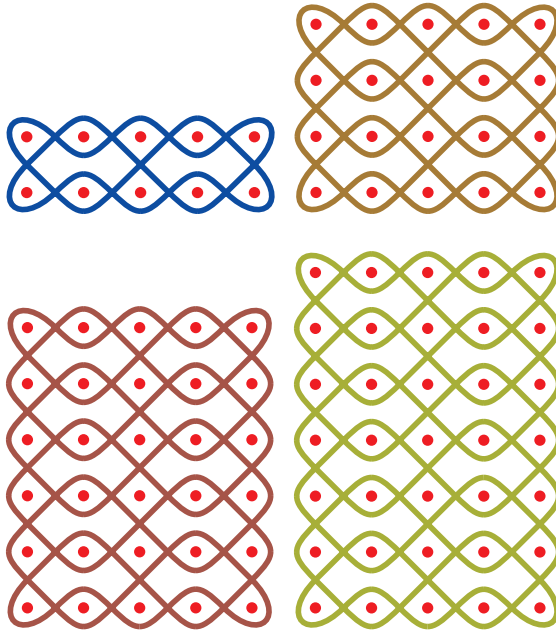




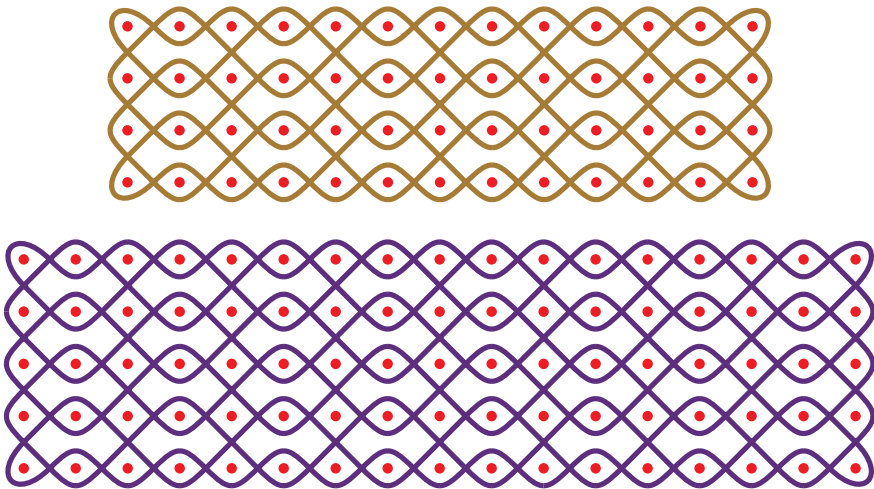
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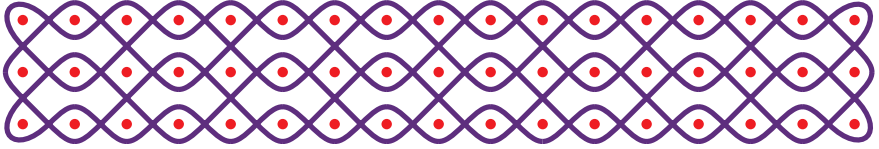
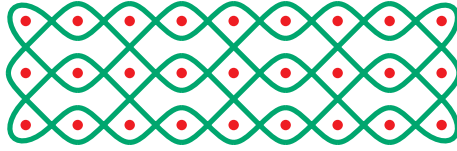
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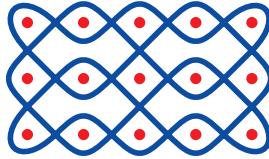
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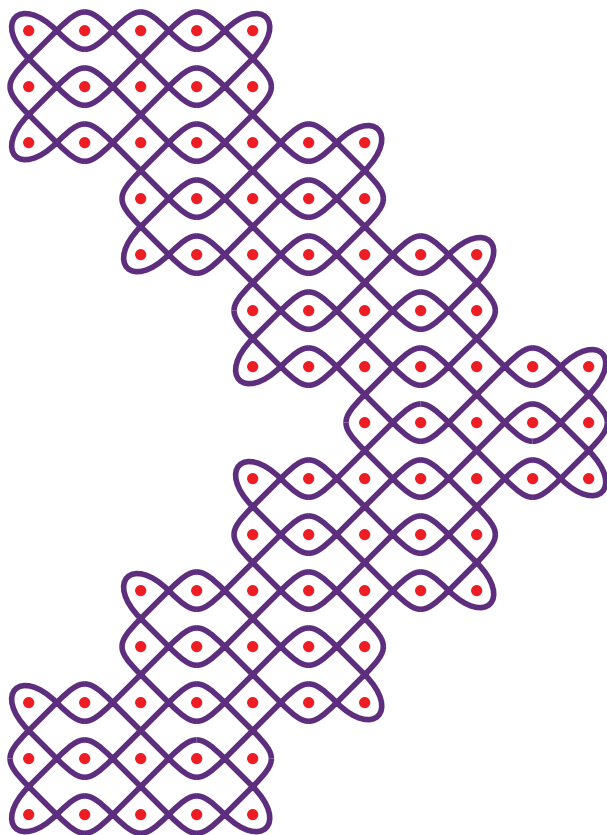


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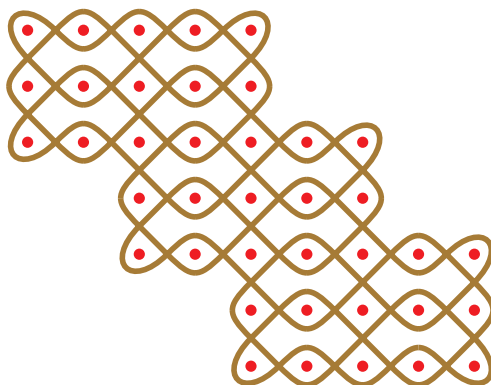


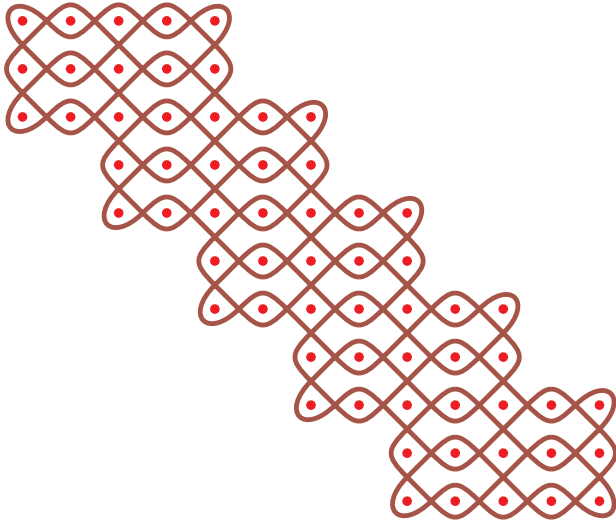
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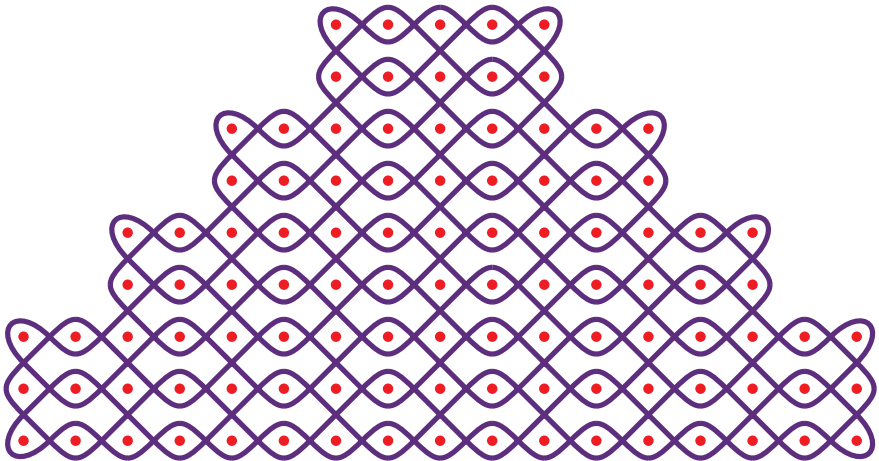
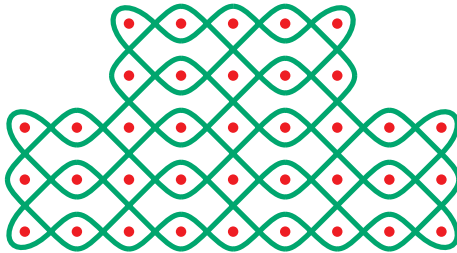


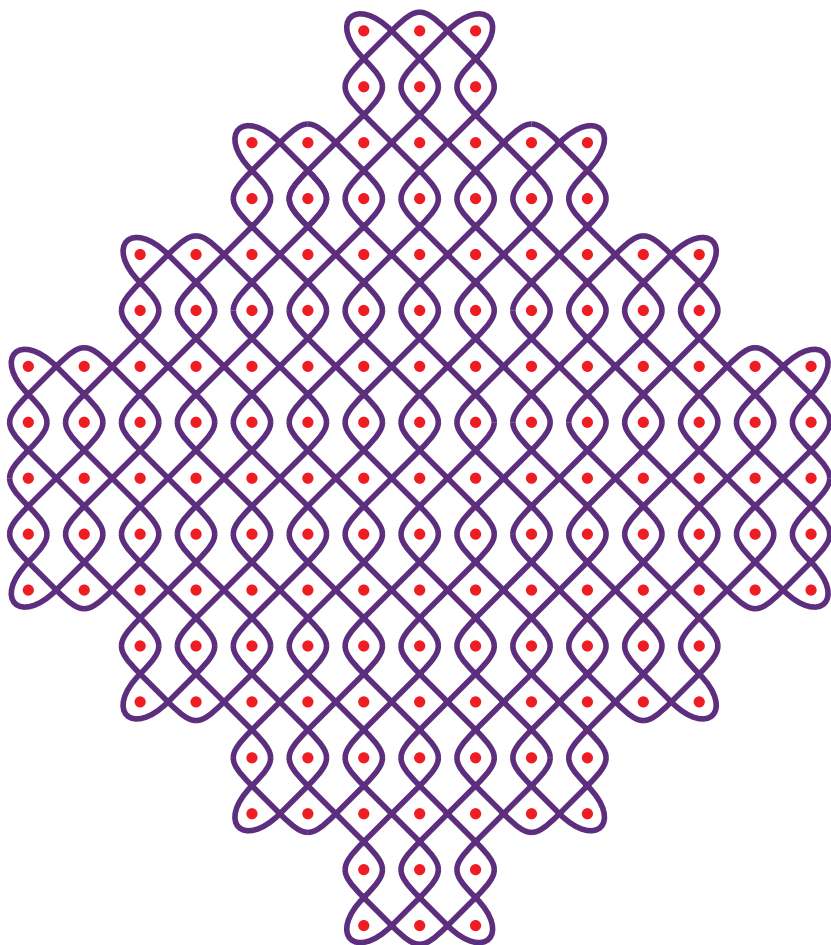
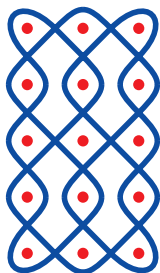
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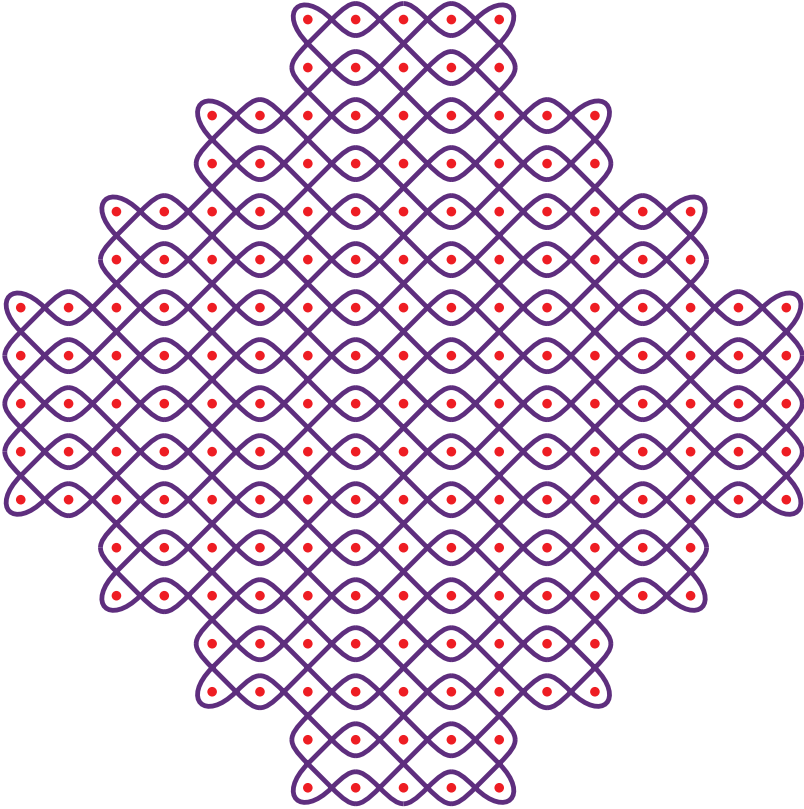
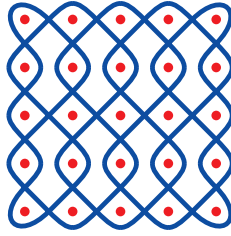


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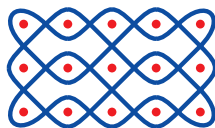


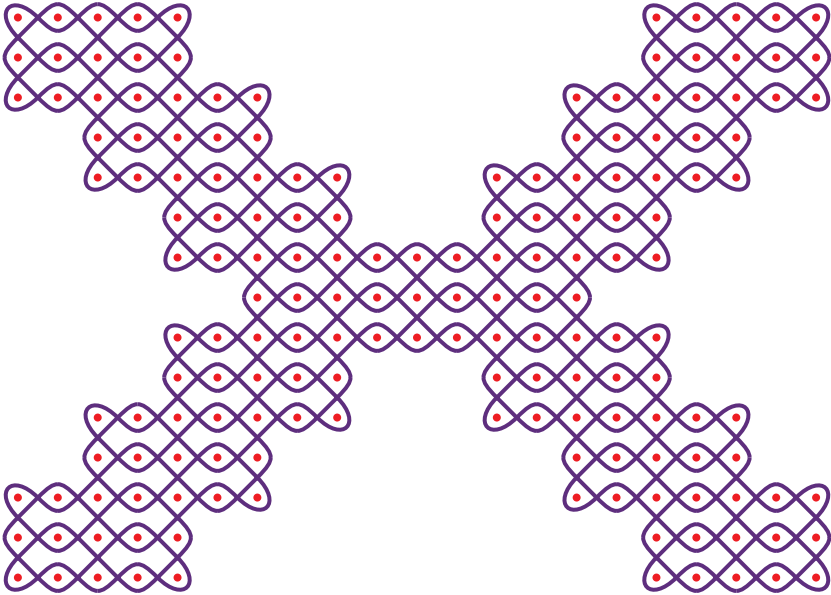


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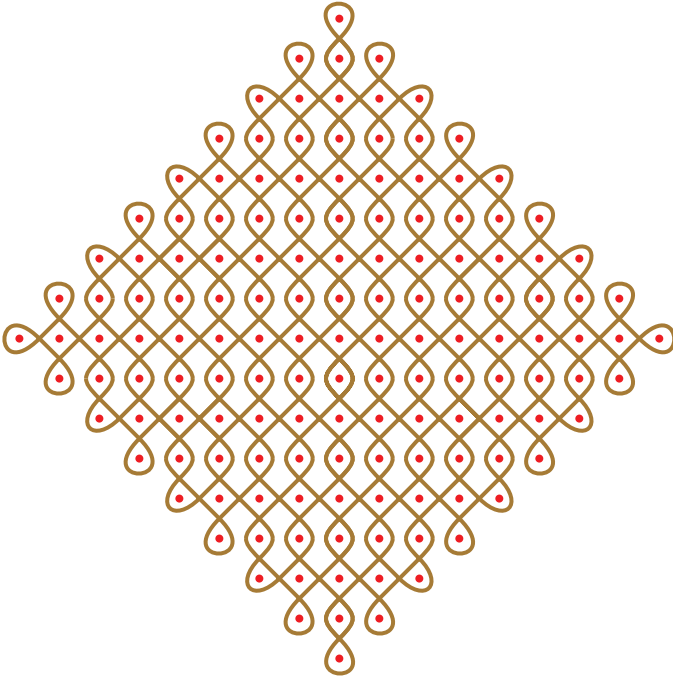


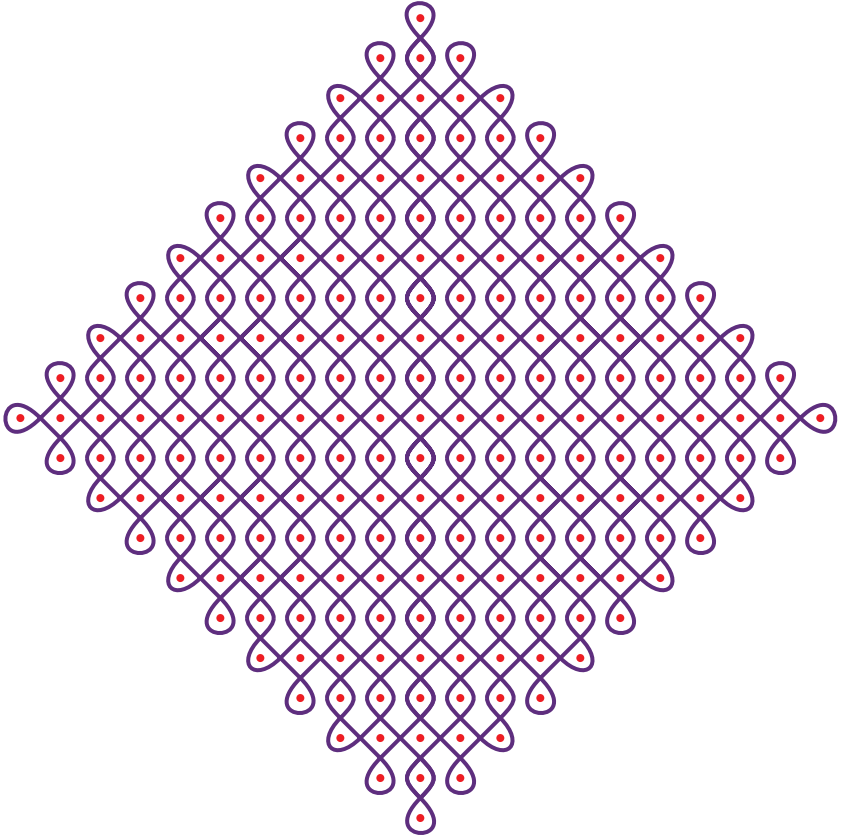
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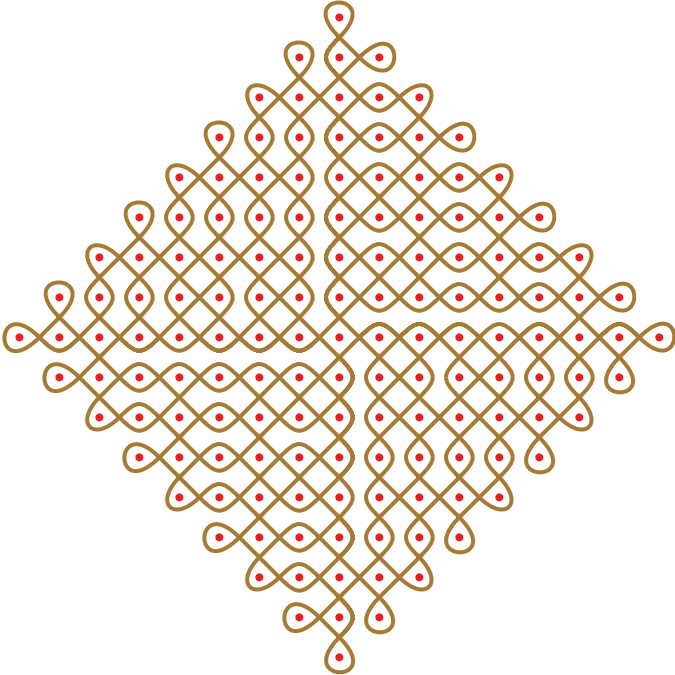


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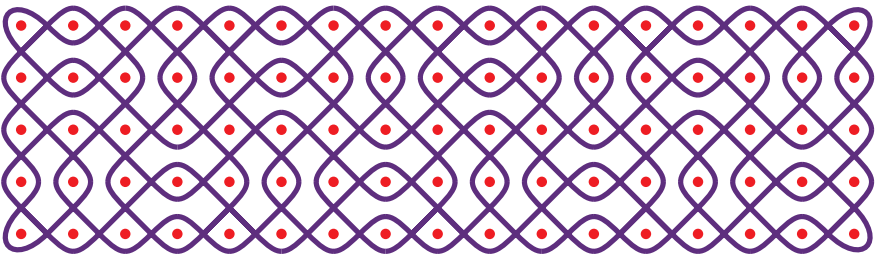
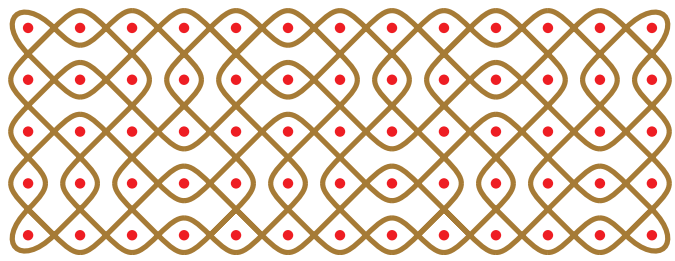


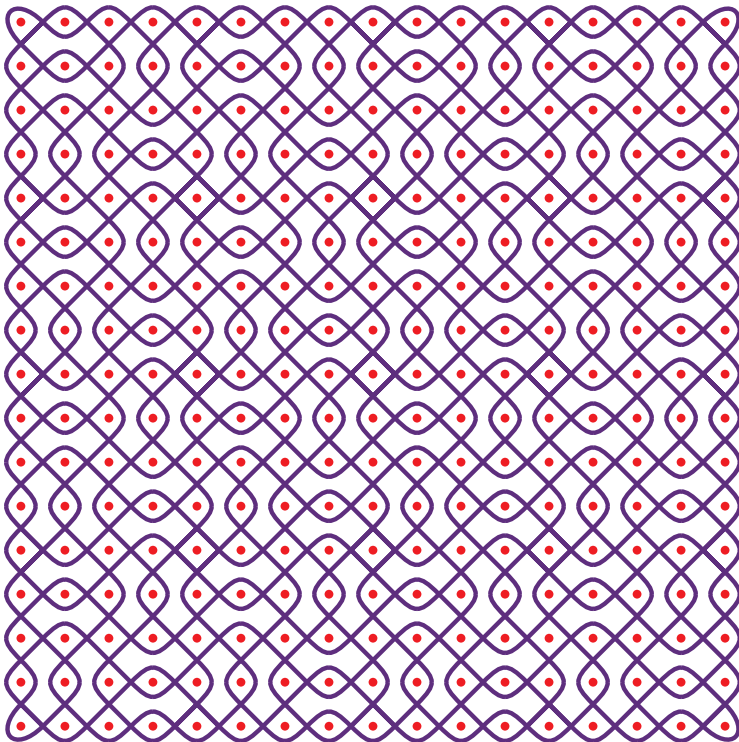
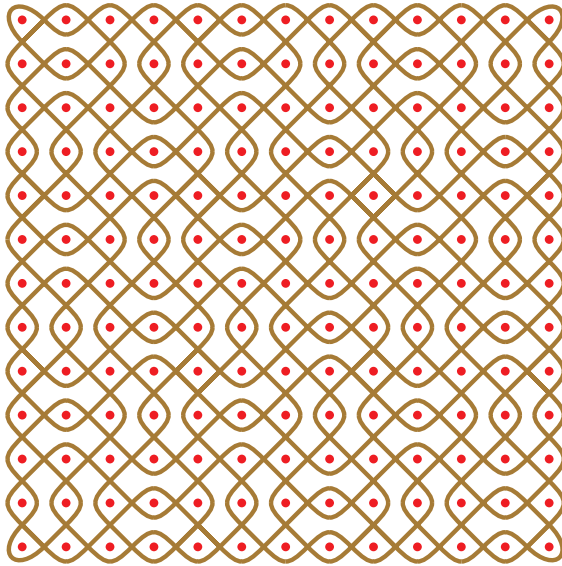


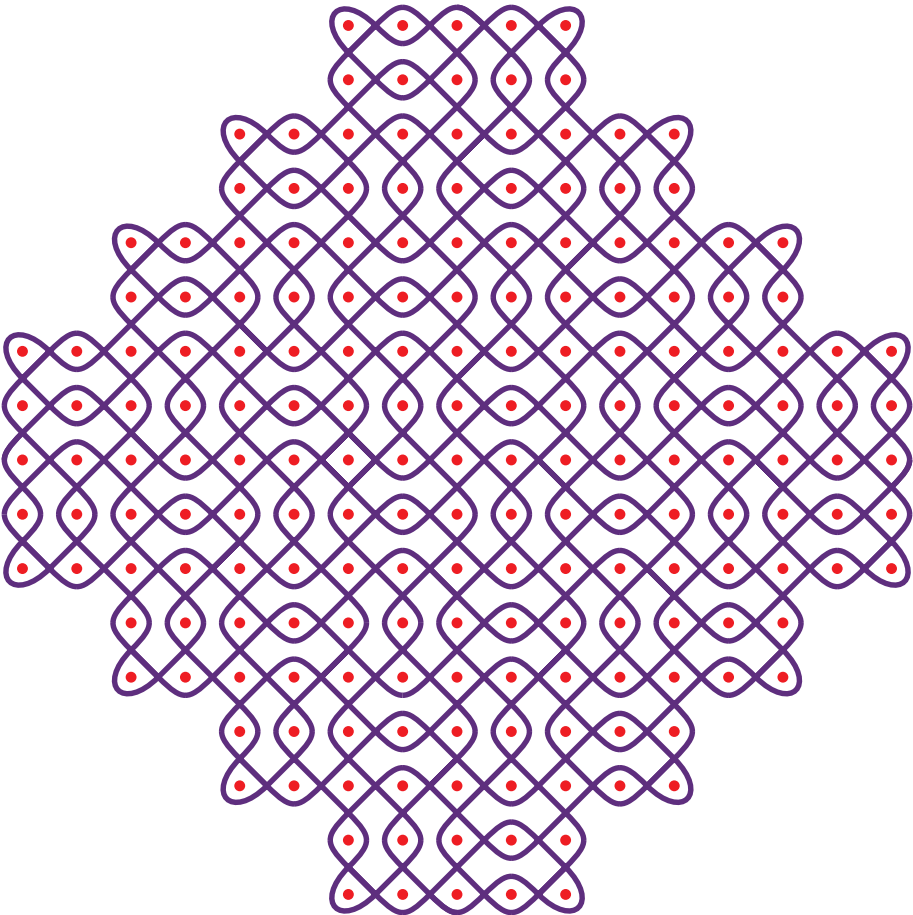
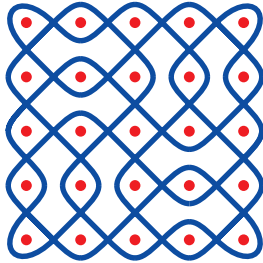
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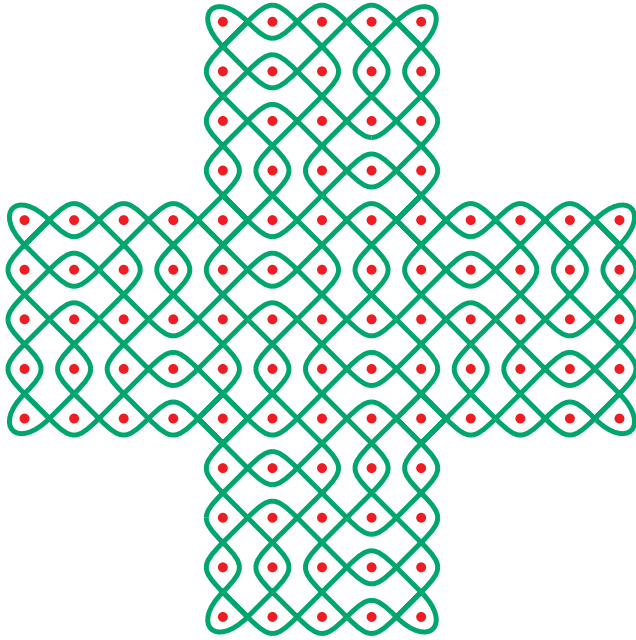
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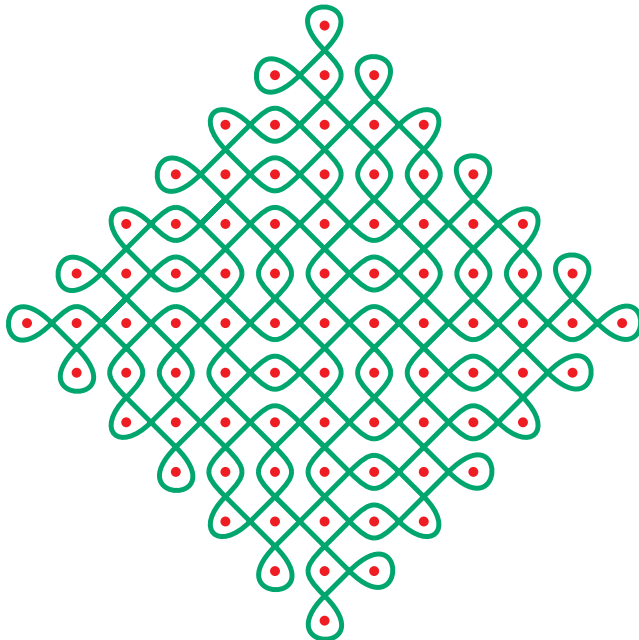




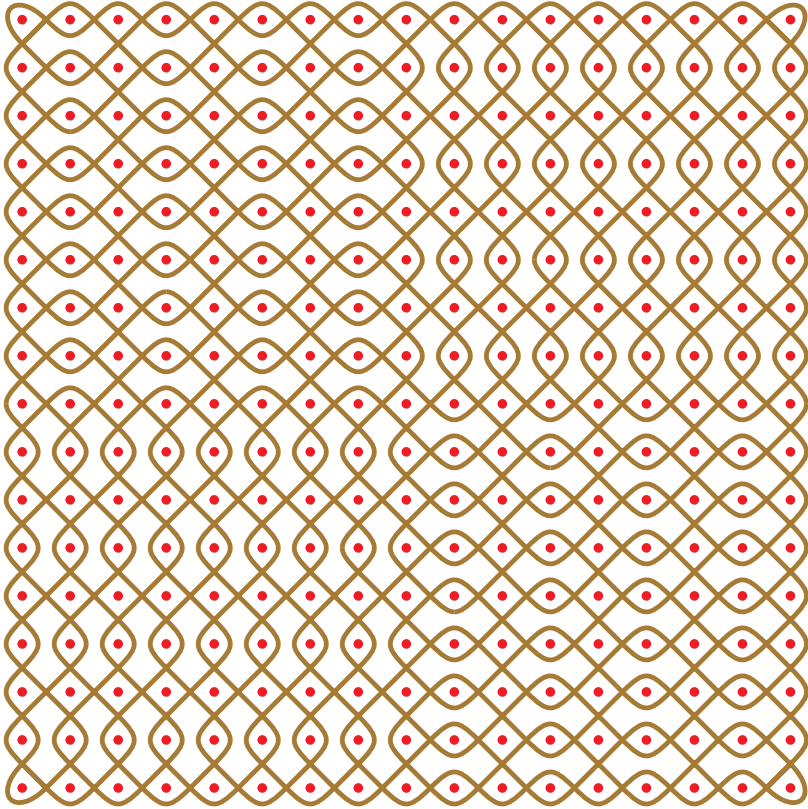
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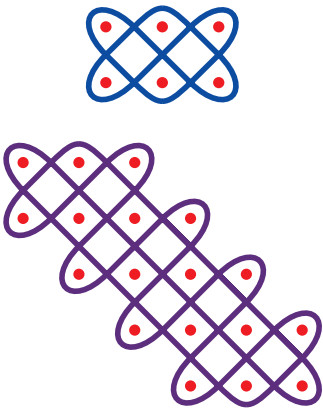
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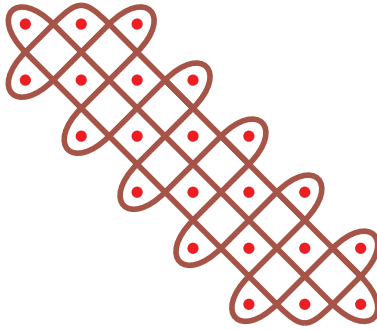


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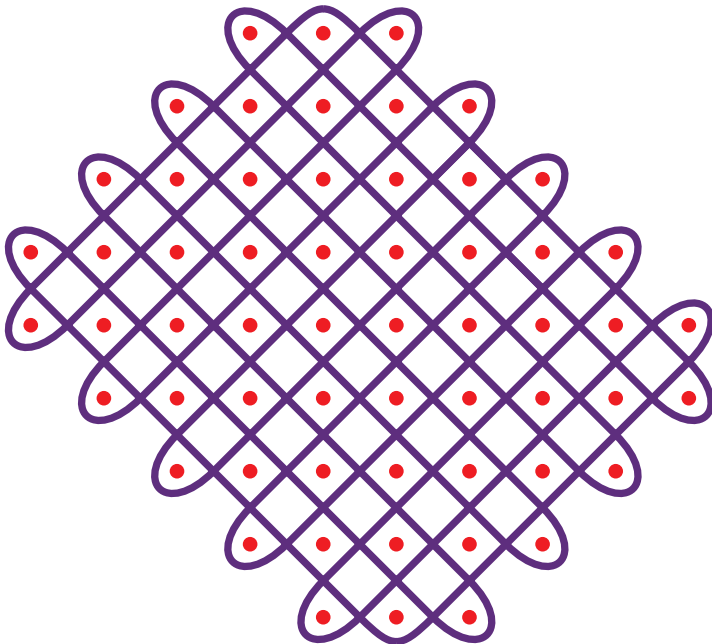
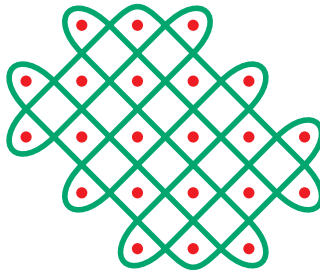


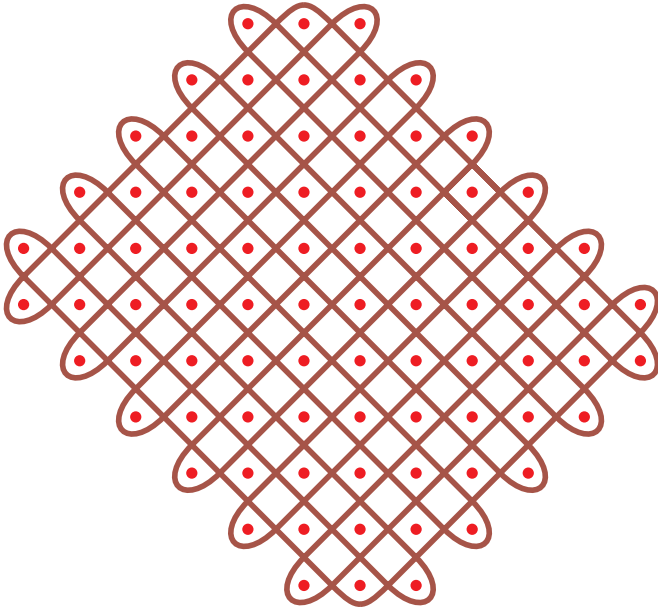
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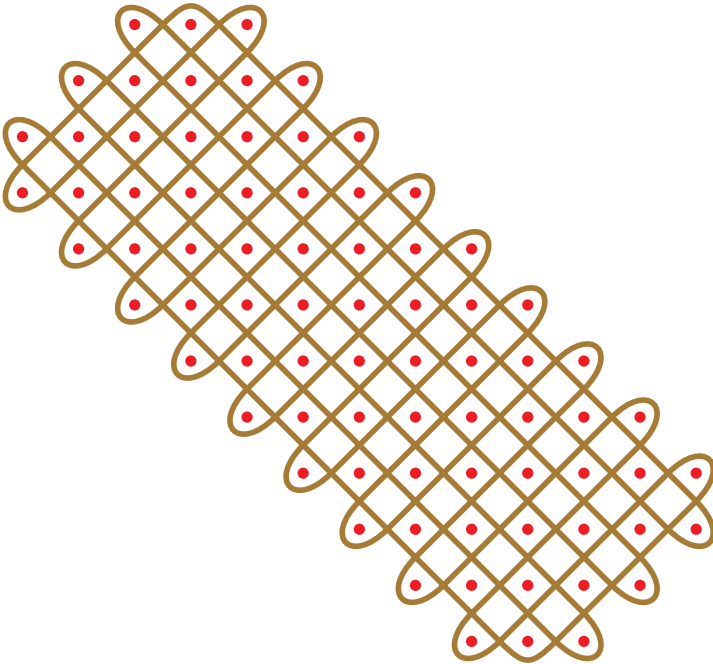


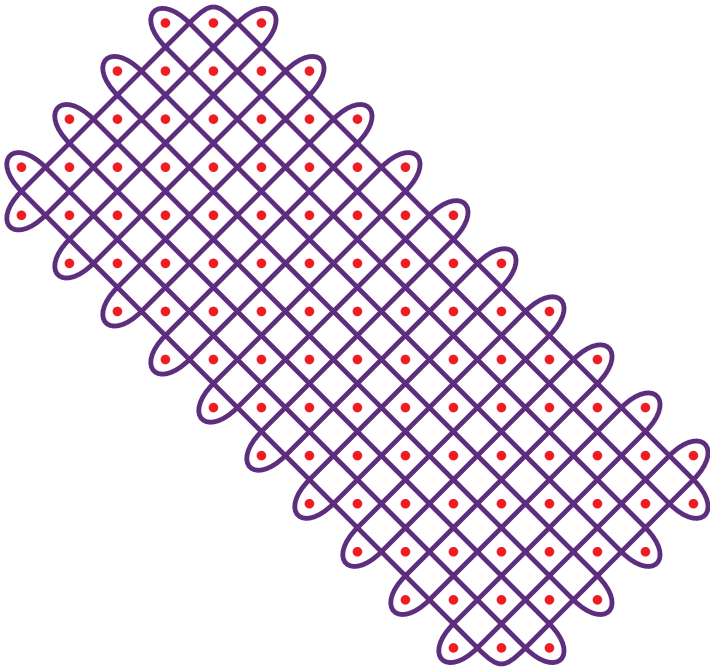
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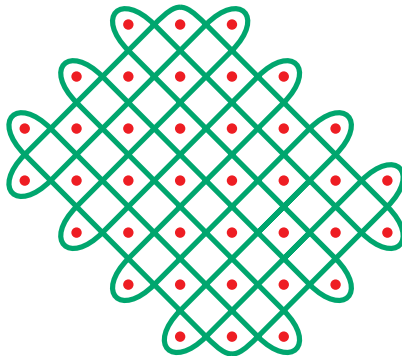
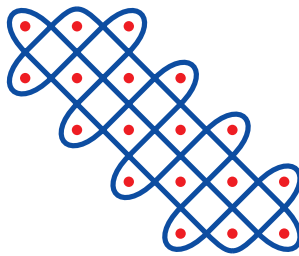


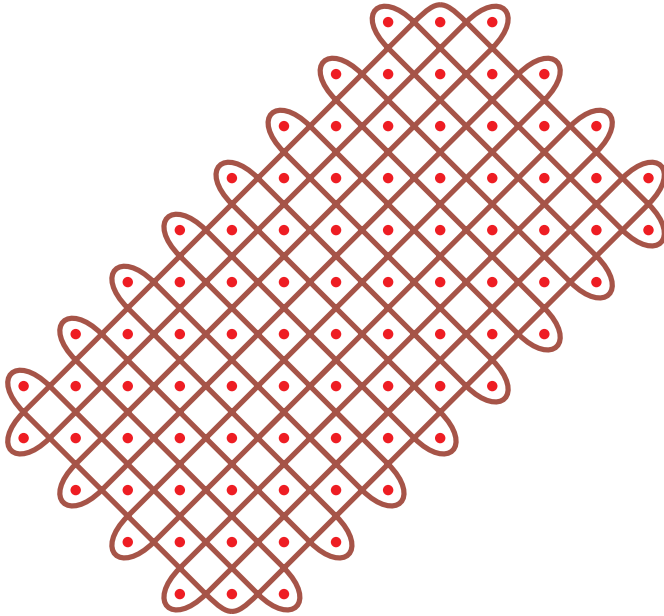
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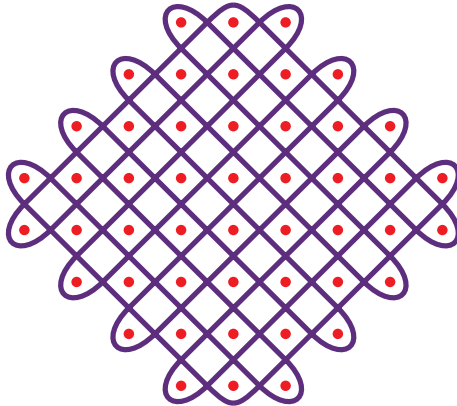
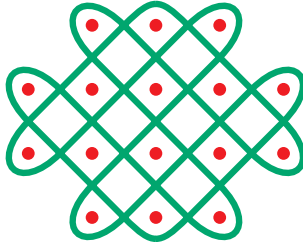


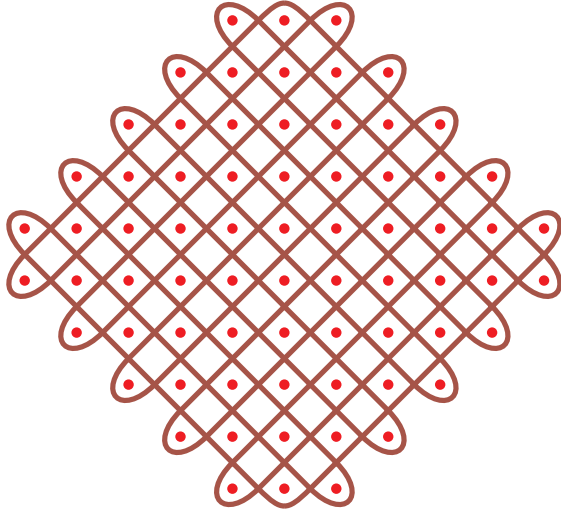
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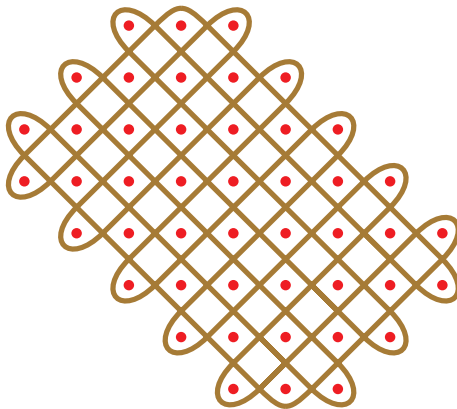


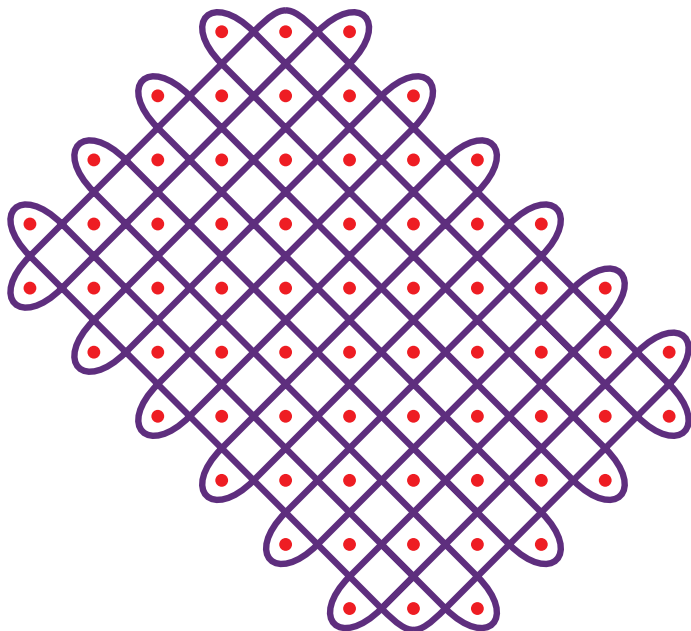
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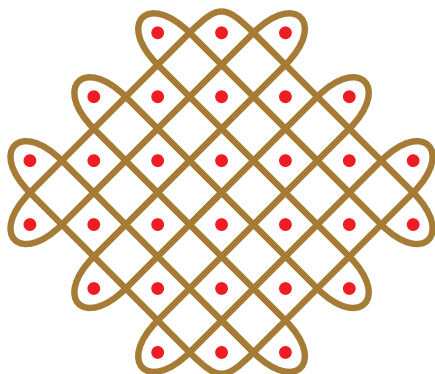
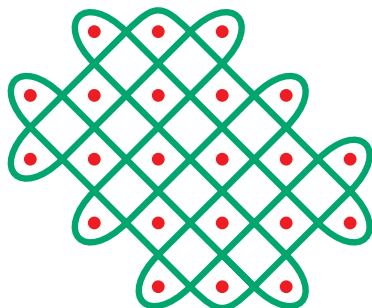


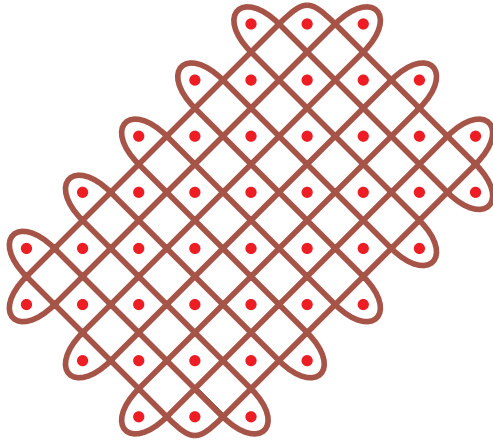
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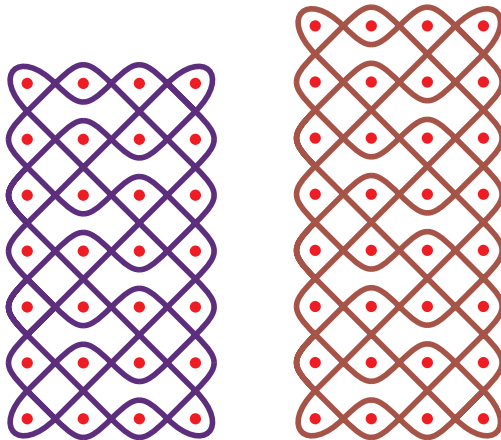


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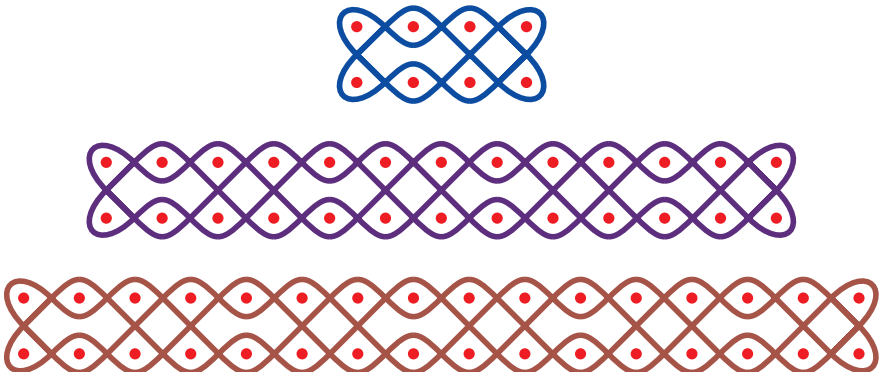




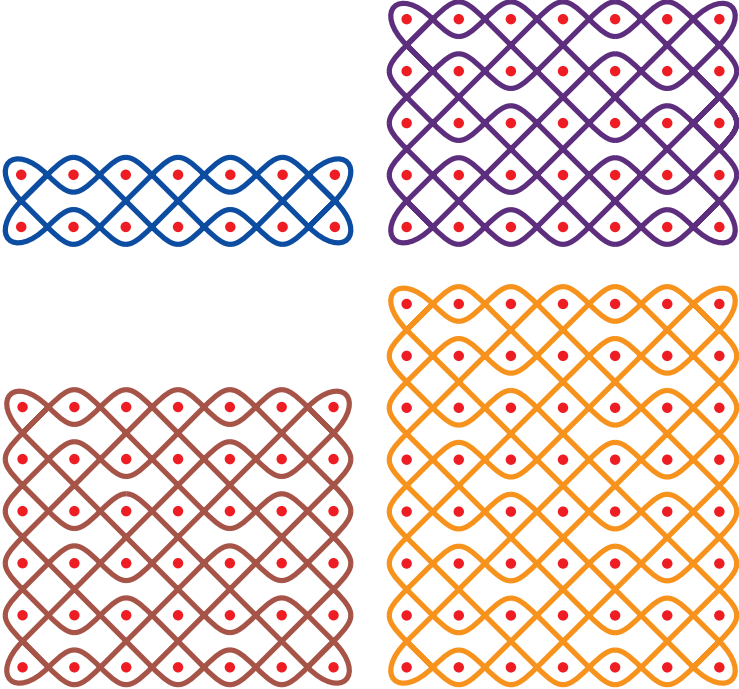
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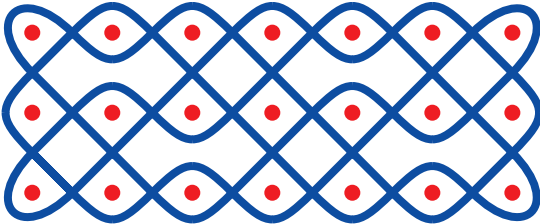
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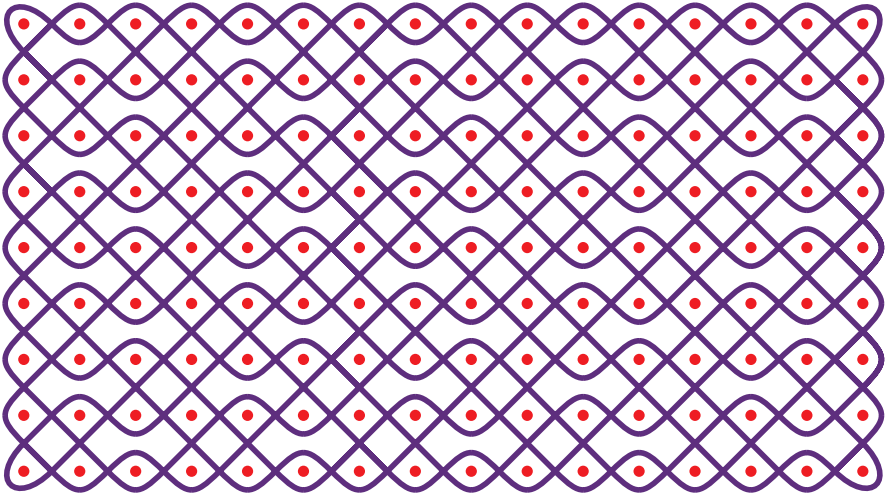


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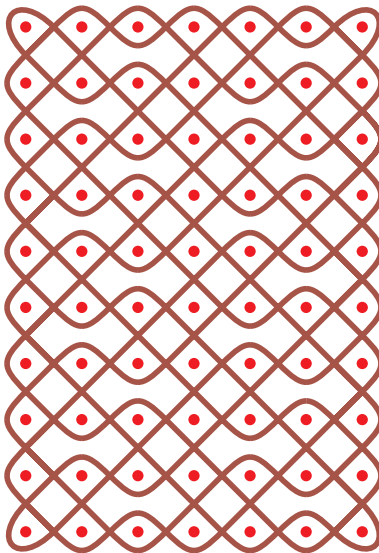
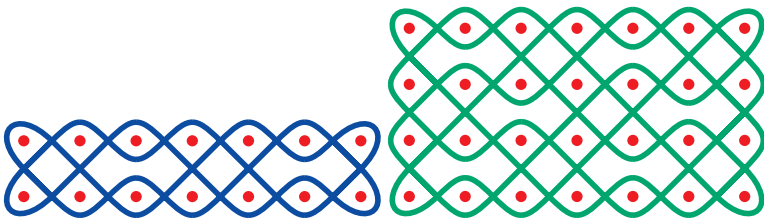


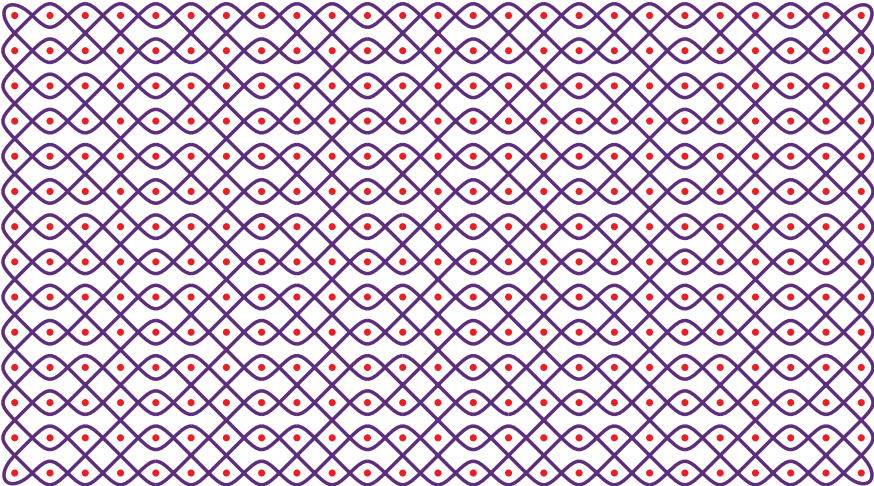
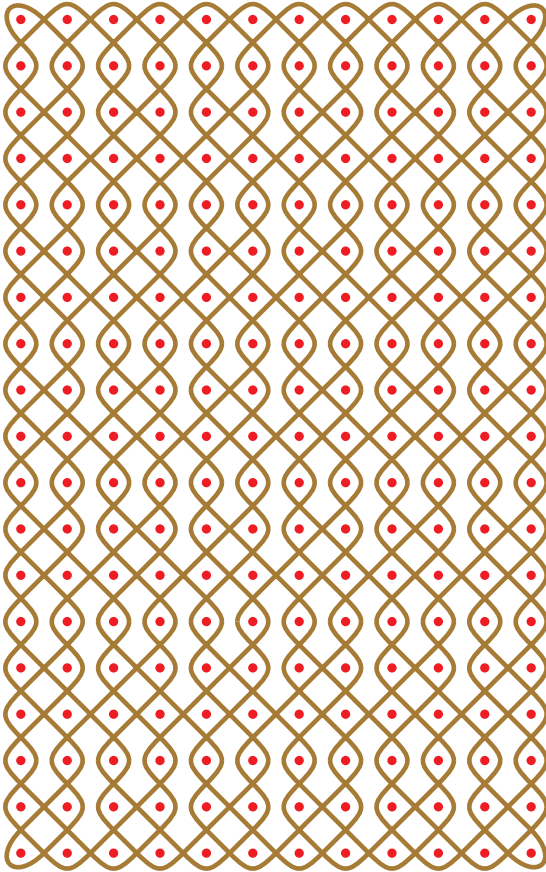
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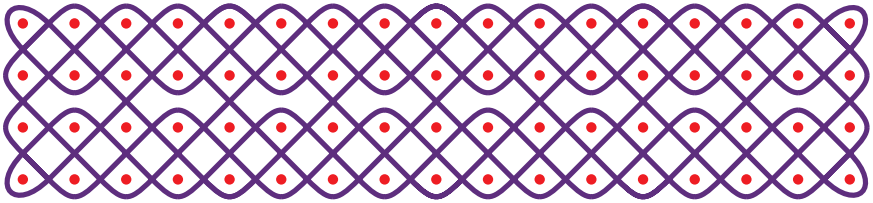
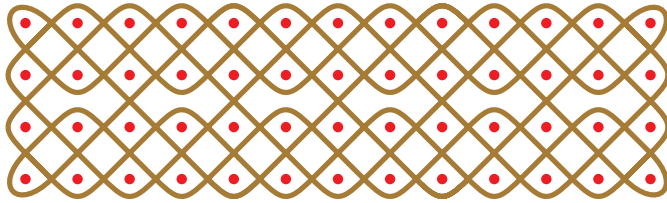


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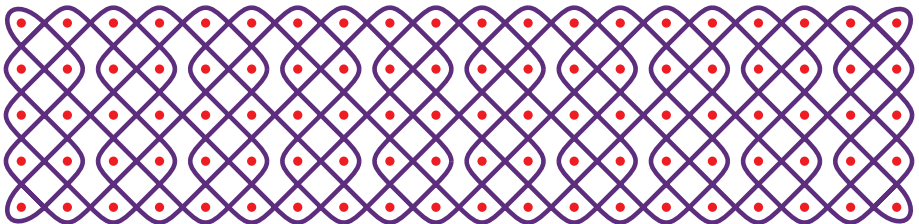
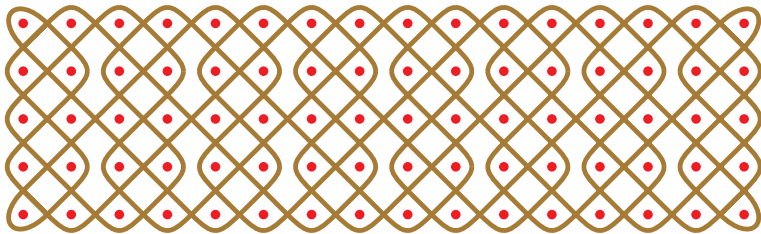


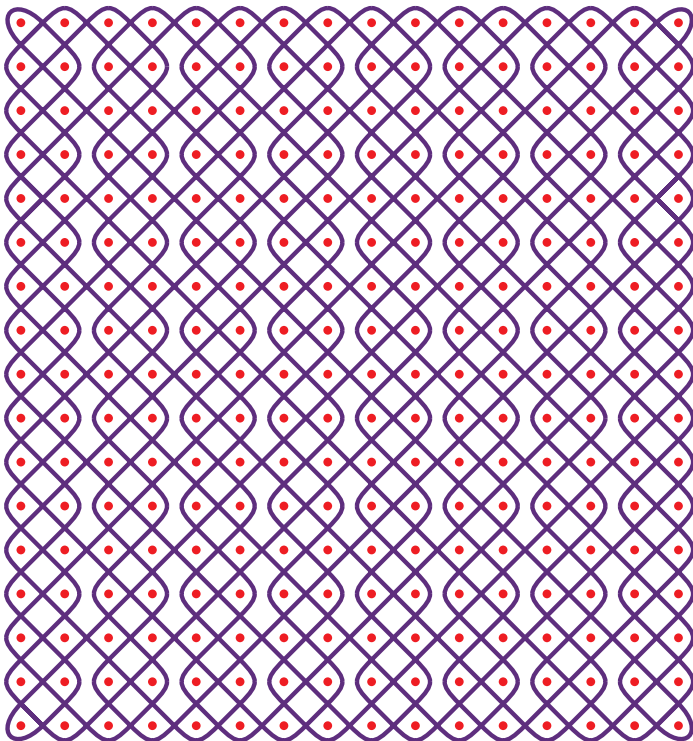
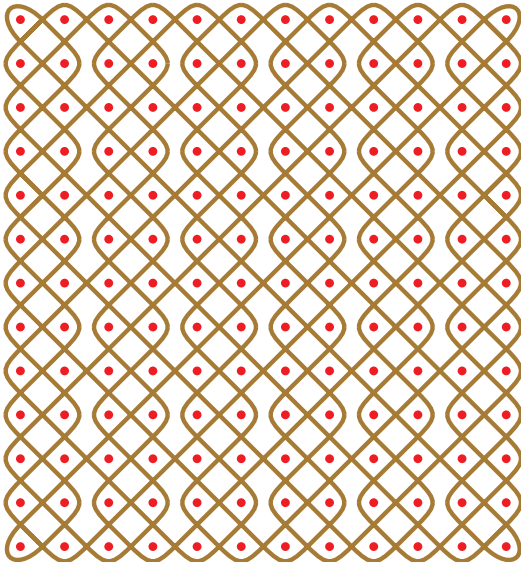


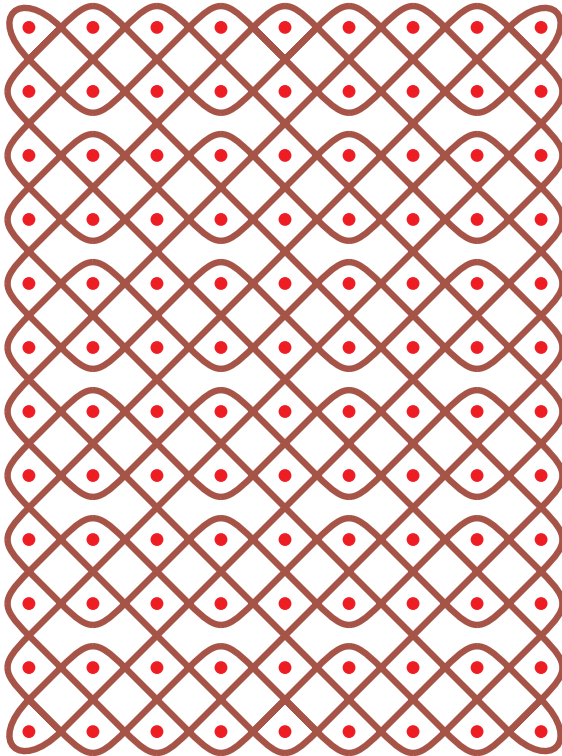
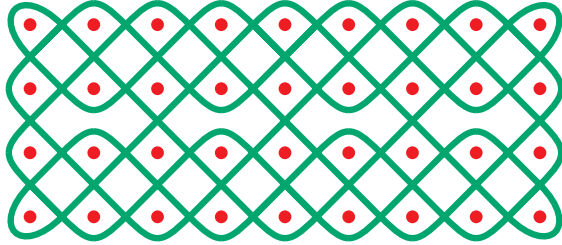
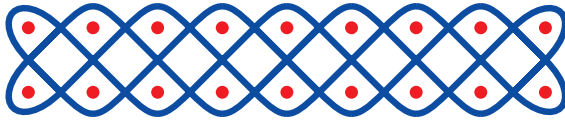
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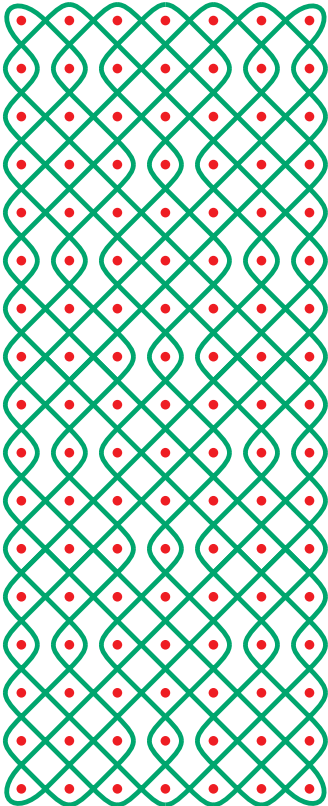
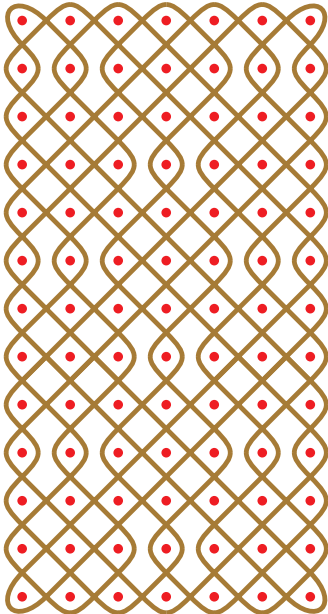
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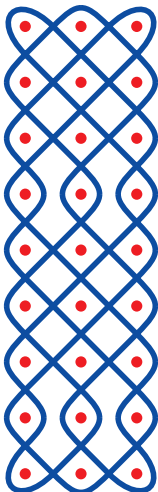


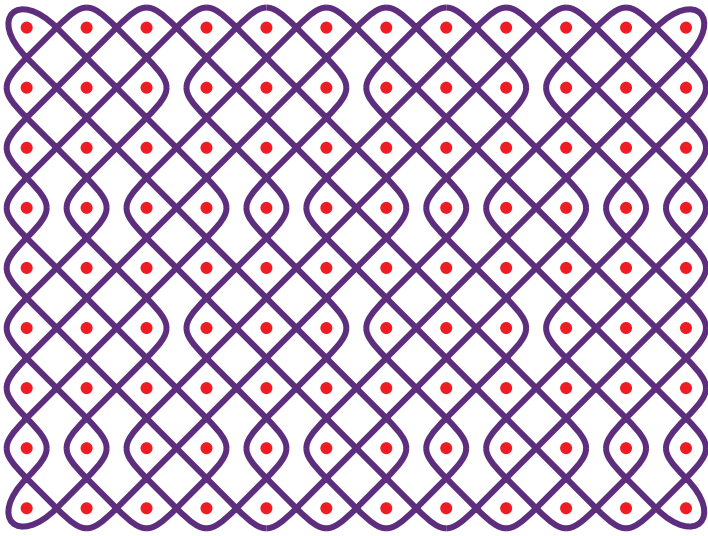


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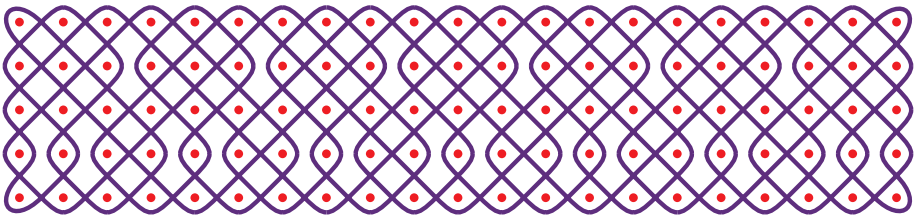
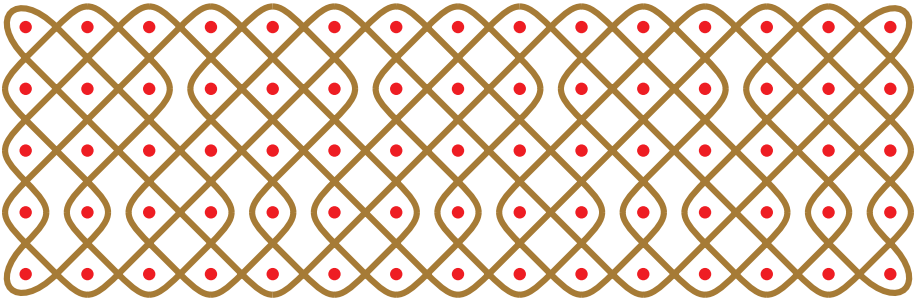


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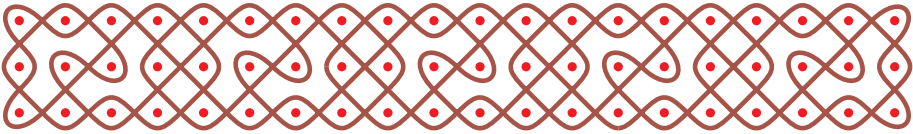
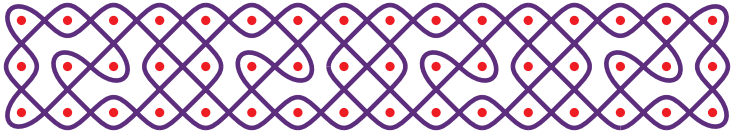




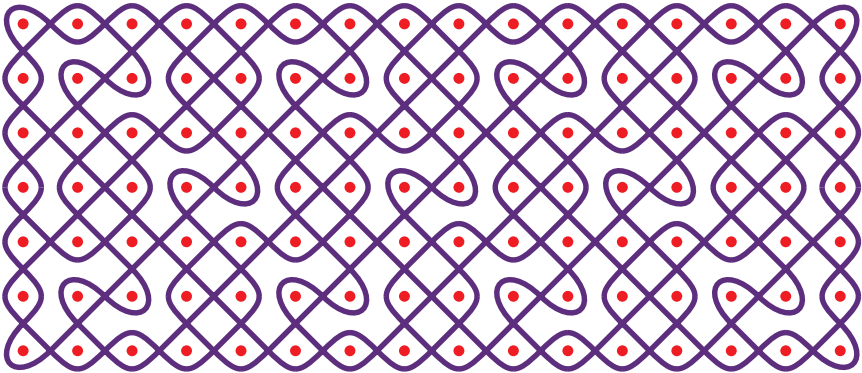
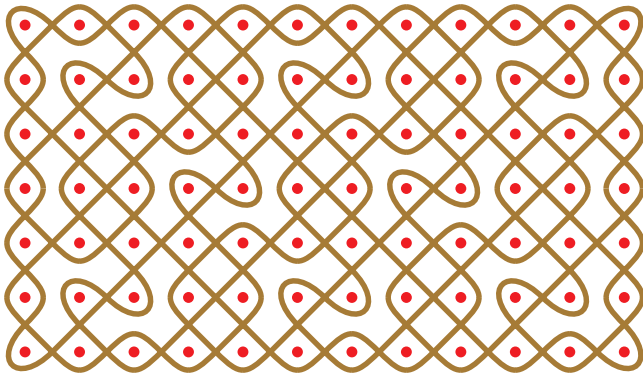
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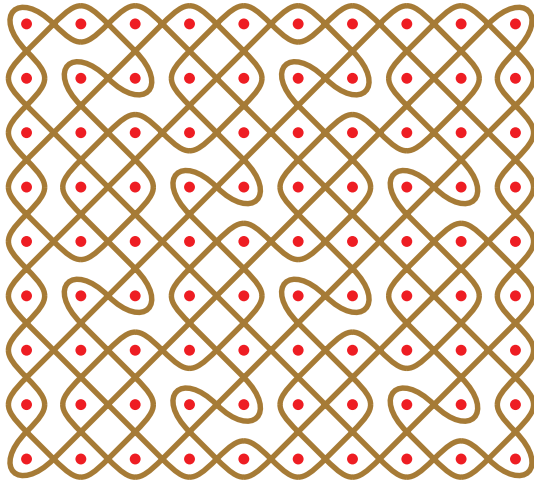
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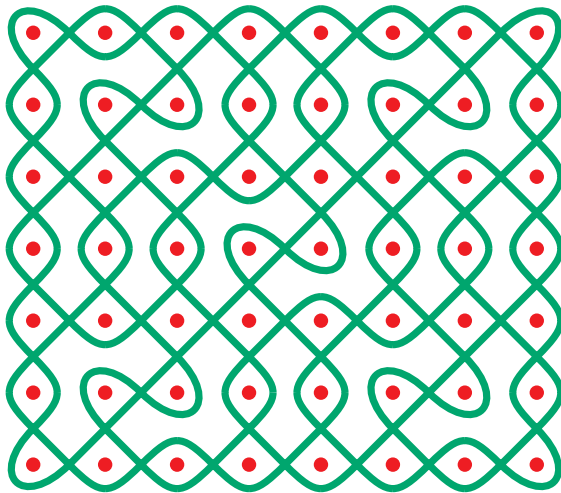
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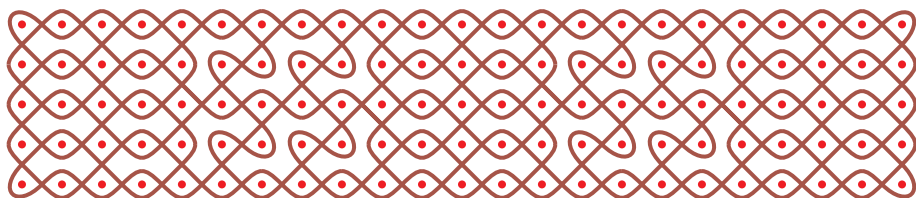
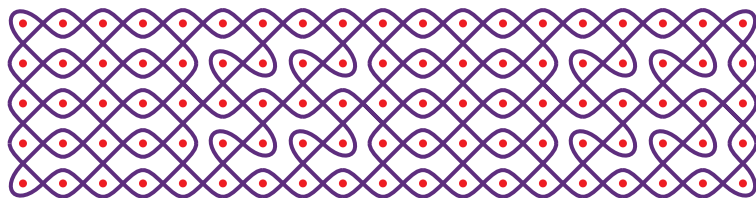
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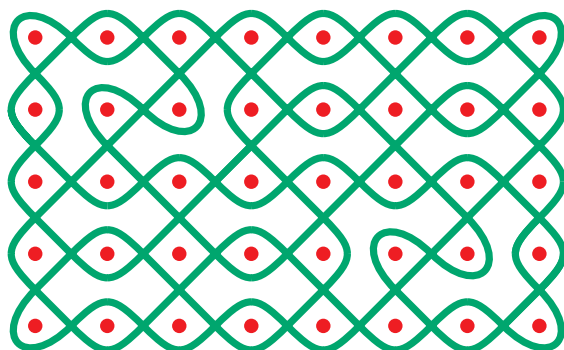
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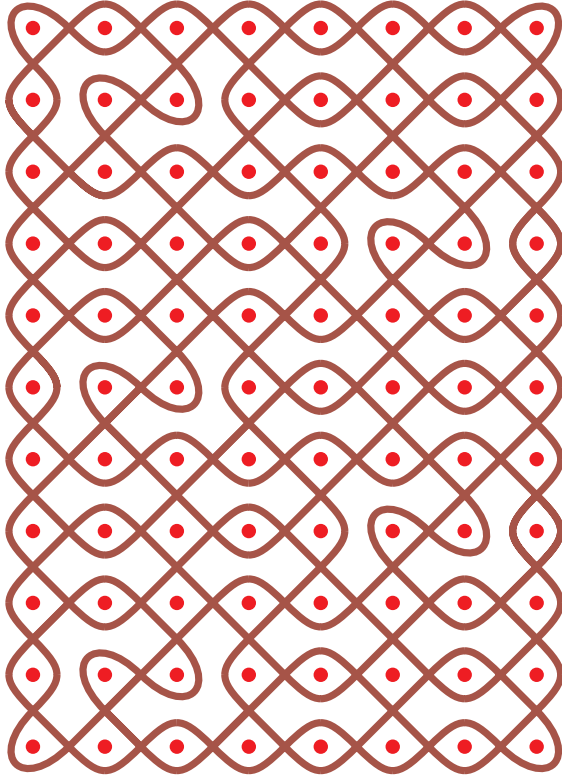


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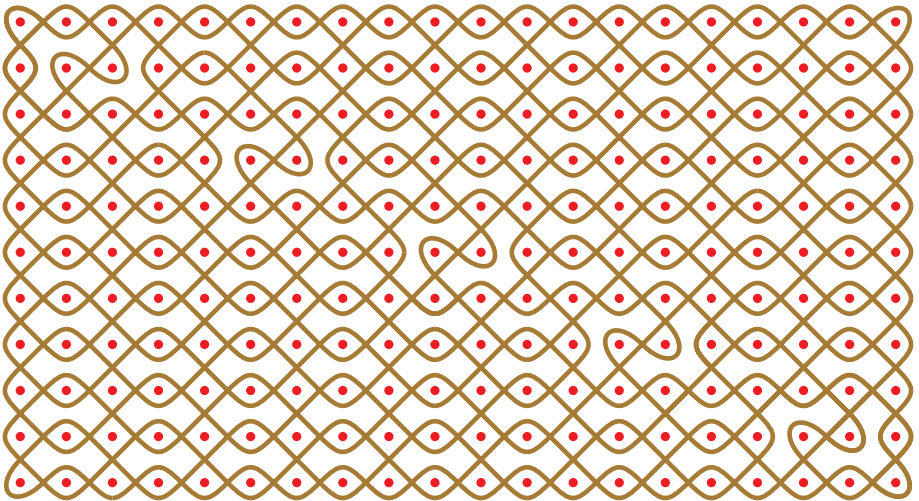


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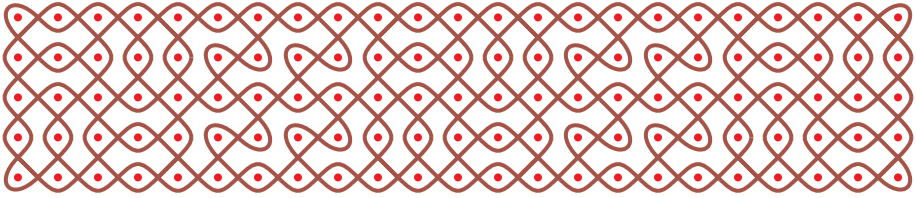
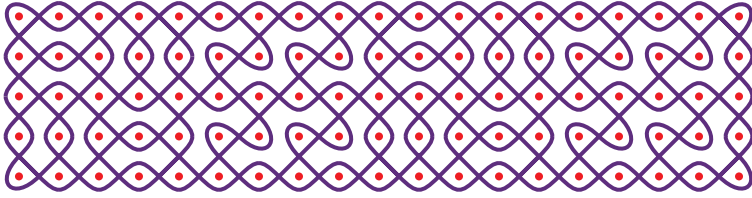




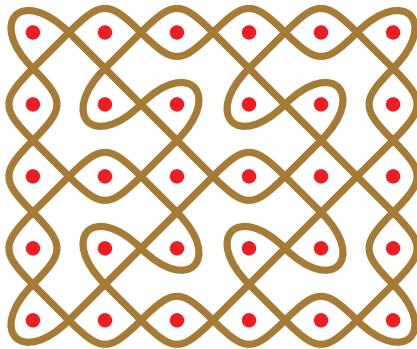
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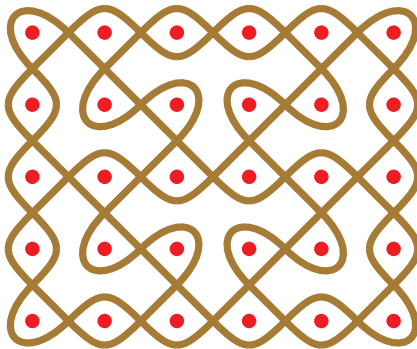
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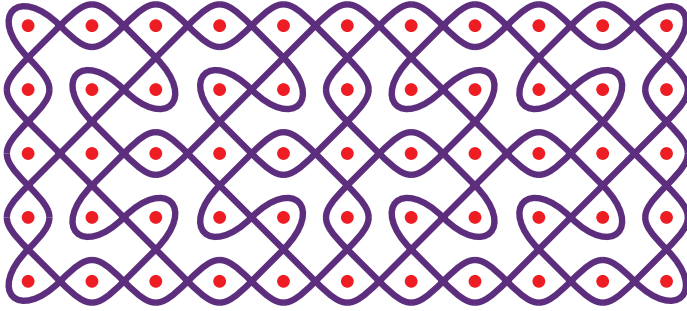
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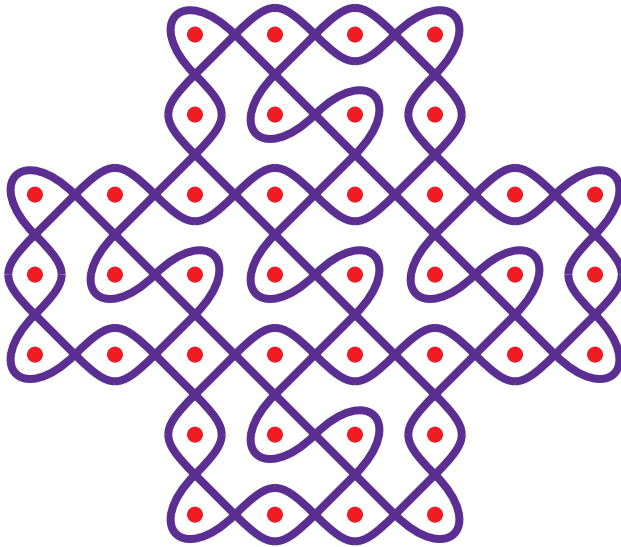
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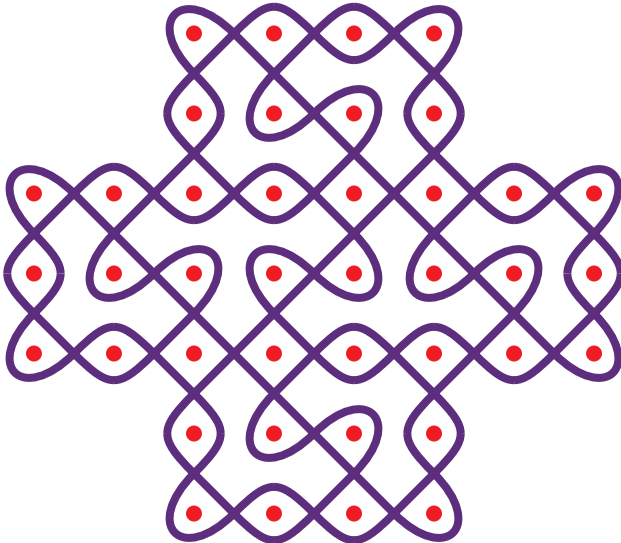
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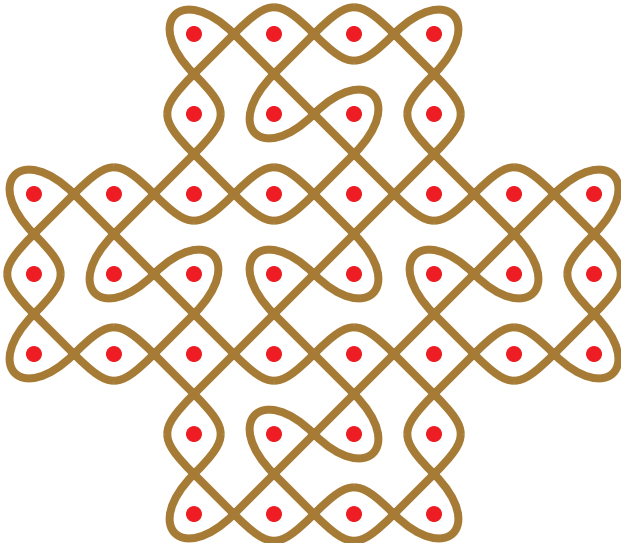
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O Autor

O professor catedrático Paulus Gerdes tem leccionado nas Universidades Eduardo Mondlane e Pedagógica (Moçambique). Desempenhou os cargos de Director da Faculdade de Educação (1983-1987) e da Faculdade de Matemática (1987-1989) da Universidade Eduardo Mondlane e de **Reitor da Universidade Pedagógica** (1989-1996). Em 2006-7, foi Presidente da Comissão Instaladora da Universidade Lúrio, a terceira universidade pública de Moçambique, com sede em Nampula.

Foi conselheiro do Ministro da Educação (2000-2005). Actualmente é conselheiro para Pesquisa e Qualidade no Instituto Superior de Tecnologias e de Gestão (ISTEG), Boane, Moçambique.

Entre as suas funções ao nível internacional constam as de Presidente da Comissão Internacional para a História da Matemática em África (desde 1986) e de Presidente da Associação Internacional para Ciência e Diversidade Cultural (2000-2004). Em 2000, sucedeu o brasileiro Ubiratan D'Ambrosio como Presidente do Grupo Internacional de Estudo da Etnomatemática.

É membro da Academia Internacional para a História da Ciência (sede em Paris). Desde 2005 é **Vice-Presidente da Academia Africana de Ciências**, responsável para a África Austral.

Escreveu diversos livros sobre geometria, cultura e história da matemática, tendo recebido vários prémios.

